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THE METHODS, CONTENT AND PURPOSE OF BIOLOGIC SCIENCE IN THE SECONDARY SCHOOLS OF THE UNITED STATES.

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In an era when the pragmatist advocates his system of philosophy; at a time when educators the country over are awakening to the fact that our secondary school is not fulfilling to the highest degree its practical efficiency; when we note in secondary education on the one hand a demand for technical and trade schools and when we see on the other hand in the university added to the already full curricula course after course leading toward practical ends, courses in applied medicine, sanitation, agriculture, forestry and the like—it is evident that time is full ripe to begin investigation of conditions within the secondary schools which bear upon this general problem of what is of most practical worth in our system of secondary education. This is preëminently the age of applied science: it is an intensely practical age; the average individual comes in daily contact with the problems of science as never before. It is self-evident that science work of an elementary sort should play an important part in the education of our youth who go into life—as the vast majority do—with no other fitting than that received in the elementary or secondary school. It was with this thought in mind that the writer began the following preliminary investigation which aims in the first place to present some statistics bearing upon the teaching of science, and especially biologic science, in the secondary schools and in the second place to suggest possible modifications in our present courses in biologic science that will make such courses a better preparation for the kind of life into which the most of our young people are launching, the active life of the thinking, doing citizen.

The following data are based upon answers made to certain questions, which follow, of a questionnaire on methods, content and purpose of biologic science in the secondary schools of the United States. Over five hundred copies of this questionnaire were sent out and replies were received from 276 schools, representing 34 different states. In every case the reply from the school represents actual and not paper conditions, and is the judgment of either the principal or head master of the school, the head of the science (biology, chemistry or physics) department, or of some qualified teacher in the department.

The schools to which the questionnaire was sent were chosen from government reports or sources of personal knowledge as being representative high schools, in large cities or urban communities. For the most part the reports come from schools located in cities of over 10,000 inhabitants; in a few schools in the central West (four out of 129) the township high school is represented.

The first chart graphically presents the tabulated answers to the question, *"What is the sequence of science courses in your school?"* Tabulation by states has been made according to the grouping suggested by T. Q. Browne, in a paper entitled, "Laboratory Equipment in Secondary Schools," SCHOOL SCIENCE AND MATHEMATICS, 1908, pp. 689-695.[1]

This table shows the total number of science courses in the four years of the high schools reporting to be unexpectedly evenly balanced: the total by years is as follows: I, 360; II, 317; III, 334; IV, 360. Of this total number of science courses, 1,371 in all, 623 are biologic (botany, zoölogy, human physiology or general biology, with scattering courses in elementary agriculture and plant physiology). This is no doubt due in part to state laws requiring the teaching of human physiology in the high school. But reducing the number of biologic courses by the total number of courses offered in human physiology, 193, it is seen that 430 courses in biologic science are given as against 166 courses in physiography, 253 courses in chemistry and 268 courses in physics.

[1]New England: Maine, New Hampshire, Massachusetts, Rhode Island, Connecticut. Middle States: New York, New Jersey, Pennsylvania, Delaware, Maryland, Dist. of Columbia. Southern States: Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Kentucky, Tennessee, Alabama, Mississippi, Louisiana, Texas, Arkansas, Oklahoma, Indian Territory. North Central: Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Iowa, Missouri. Rocky Mountain: North Dakota, South Dakota, Nebraska, Kansas, Montana, Wyoming, Colorado, New Mexico, Arizona, Utah, Nevada, Idaho. Pacific: Washington, Oregon, California.

SCIENCE COURSES GIVEN IN HIGH SCHOOLS	NUMBER OF SCHOOLS	FIRST YEAR OF HIGH SCHOOL								SECOND YEAR OF HIGH SCHOOL								THIRD YEAR OF HIGH SCHOOL								FOURTH YEAR OF HIGH SCHOOL								
		Botany	Biology	Human Physiology, Hygiene	Introductory Science	Physical Geography	Zoölogy	Physics, Chem., Astr., Geol.		Botany	Biology	Human Physiology	Chemistry	Physical Geography	Physics	Zoölogy		Botany	Biology	Human Physiology	Chemistry	Physical Geography	Physics	Zoölogy	Astronomy & Geology	Botany	Biology	Human Physiology	Chemistry	Physics	Physical Geography	Zoölogy	Astronomy	Geology
Maine	1									1		1											1					1					
New Hampshire	2	1				1	1	1		2	1				2			1			1		2	1		1		2	1		1		
Massachusetts	31	14	3	10	3	4	4	7		8	4	5	1	3	13	10		5	3	12	17	1	20	2	4		5	4	13	22	11	3	3	2	1
Vermont	1	1				1				1	1					1							1							1			1	1
Rhode Island	4	3					1	1		1			1		3	1				1	2				1						2		1	1
Connecticut	13	1		1	1	5		1		5	2	1	1	3	2	3					3	1	4	1		1	1	1	4	3	1		2	2
New Eng. States, Total.	52	20	3	11	4	11	6	10		18	8	7	3	6	18	17		6	3	13	23	2	28	4	5		7	5	14	30	15	6	4	6	5
New York	39	6	28	35	1	2	2			5	1	3		10	1	6		3			12		29	2			7			26	11	5	5		
New Jersey	11	2	1	4		3	2			4	1	3		1	1	2		1			1		7	2			1		1	7	1			1	1
Pennsylvania	25	7	1	7	1	5	2	1		6	1	3	2	2	1	5		2	1	1	6	1	7	5	1		1		2	8	14		2		
Delaware	1			1																														
Maryland	3	3				3									3								3						8					
Dist. of Columbia	3					3				1	1		2		2				2		3		3	1		1	1		3	3		2		
Middle States, Total.	82	18	30	47	2	16	6	1		16	4	9	4	13	5	16		6	3	1	22	1	49	10	1		10	1	3	47	29	5	9	1	1
Virginia	3	1				1	1			1				1		1					1		1							1	1			
Georgia	1	1						1													1													
Kentucky	6			4		1	1			3				1		2		1			1		4				1			2	1	1		1	
Tennessee	2			1		1					1												1											
Louisiana	1	1				1									1								1						1					
Oklahoma	1					1				1					1						1									1				
Southern States, Total.	14	3		5		5	2	1		5	1		2		5			1			4		7				1			4	3	1		1	
Ohio	23	8	1	8	1	9	6			10	2	3		8		7		1	1		9	1	14	1	2		5	1	6	15	7				1
Indiana	13	6				3	1			5		1		5		2		2			3		7	1					2	6	3	1			
Illinois	28			20	1	17	2			13	2	1		2		11					3		15							15	4			3	2
Michigan	13	6		1		6	1			6	1	5		3		9				2	10	1	2							1	9				1
Wisconsin	10	4		5		6	2			6		4		1		4		1		1	8		1	2						2	9		1		
Minnesota	5	1		1						1				4		1		3					5				3			5	2				
Iowa	7	5				4	1			1	1	1		1		3					2		4	1					3	4	3			1	
Missouri	8	4	1	4		1					1			2	2			1			2		6	1						6		3			
N. Cent. States, Total.	107	34	2	39	2	46	13			42	7	15		26	2	37		8	1	3	37	2	54	6	2		8	1	11	54	37	4	1	4	4
North Dakota	2			2		2		1		2						2		1			2	1					1		1		2				
Nebraska	2	1			1					1		1				1							2							2					
Colorado	2					2				2						1				1			2	1					1	2					
Utah	1					1				1						1							1							1					
Kansas	4			1		4				3		1	1			1		2		1	3		2	1						1	2				
B'ky Mt. States, Total.	11	1		3	1	9		1		9		2	1			6		3		2	5	1	7	2			1		2	6	4				
California	6		1			5				2	3			2		2		2		1	3	1	1	2			2		3	3	2	1	1	1	
Washington	2					2				2		1				1				1			2							2					
Pacific States, Total.	8		1			7				4	3	1		2		3		2		2	3	1	3	2			2		3	5	2	1	1	1	
GRAND TOTAL	276	76	36	105	9	94	27	13		94	23	34	8	49	25	84		26	7	21	94	6	148	24	8		29	7	33	146	90	17	15	13	10

This growth in biologic science appears to have been completely overlooked in the recent reports of the commissioner of education. See Report of the Commissioner of Education, 1906, Vol. II, pp. 699-703.

The following data compiled from this report are of interest here. The percentages computed are for all students graduating from public and private secondary schools in the United States.

Total percentage preparing for college:

	1895-96	1905-06
Classical	10.05	5.65
Scientific	7.16	4.56

All students graduating from public and private secondary schools in the United States.

Number of graduates reported taking various subjects:

	1889-90	1895-96	1905-06
Astronomy		4.79	.91
Physics	22.21	22.77	15.27
Chemistry	10.10	9.15	6.52
Physical Geography		23.89	20.97
Geology		5.00	2.31
Physiology		29.95	20.36

Latin, French, German, Algebra, Rhetoric, English Literature, History (other than United States) show decided increases; Civics shows a decrease.

There has been a natural falling off in the attendance upon other courses in natural science in the secondary schools, probably due to the student's election of biologic science. This has given rise to more than one printed statement to the effect that the number of students taking science in the secondary schools is decreasing.[2]

A glance at the table shows (1) that physiography (earth study) and biologic science tend to appear early in the curricula of the secondary school, physics and chemistry coming later. In the eastern part of the United States biologic science is mostly taught in the first year of the high school, in the central states and the far West biologic science is replaced to a large extent by physiographic science, the former subject moving up as a second year science.[3]

[2]The possible sources of error in compiling this data arise not so much from lack of definite information as from the interpretation of the answers made. In the summary made above courses in biological science and, to a somewhat less extent, courses in chemistry, physics and physiography are offered in more than one year of the high school, but may be given as a single course open to pupils in the several years of the high school. Such courses obviously must be counted as given in the years specified by the answerer of the question. Thus although practically but one course is given several may be credited.

Botany, zoölogy and human physiology may be given during a single year in the high school, forming a more or less connected course, which should be credited as a single course but which appears in the table as three separate courses.

Botany, zoölogy and human physiology may be given during the same year as one continuous course but not be so specified.

In their anxiety to give biological information teachers may have omitted to report on all other sciences.

It is better, then, for us not to consider these data as exact in the strictest sense, but to use them as an indication of the trend of modern high school science teaching.

[3]This latter condition agrees with the report of Prof. O. W. Caldwell, "School Science and Mathematics," 1909, pp. 581-597. His report, based upon data compiled from a questionnaire sent out to teachers largely replying from the Central and Western States, is as follows:

All biological work in the first year of high school, 7 schools
All biological work in the second year of high school, 21 schools
All biological work in the third year of high school, 2 schools
All biological work in the fourth year of high school, 2 schools
Biologic work in more than one year of high school, 16 schools

(2) The place of physiography appears to be undetermined. In the Central, Rocky Mountain and Pacific divisions this comes as a first year science or as an option in the later years of the secondary school. In the East and South it appears later (usually the second year) or may be omitted entirely from the course of study. Pedagogically it would seem that earth science, the broader science of environment, should precede the more specialized sciences. But a practical reason exists in the extant laws requiring the teaching of human physiology in the first year of the high school and for the teaching of the biological sciences early in the course. This matter will be referred to in more detail later.

(3) Considerable divergence of opinion regarding the sequence of physics and chemistry is evidently present in the minds of educators. The position accorded to them is in a large proportion of schools in the third and fourth year. In the Middle and Pacific states physics appears in the third and chemistry in the fourth year. In the New England and North Central states the sequence is less marked, two distinct groups of schools existing, the one group giving physics in the third year, the other group chemistry in the third year.

(4) The elective system in science appears to have reached its maximum in the New England states, Massachusetts allowing the most election and offering the greatest number of courses. The Central states allow considerable election in the first two years fixing definitely on physics and chemistry in the later years of the secondary school.

(5) Sequence in science courses is most marked in certain states in the Eastern, Middle, North Central, Rocky Mountain and Pacific groups. This is in part due either to state laws, the dictates of state educational departments or to the domination of colleges in requiring certain sciences for college entrance.

One noteworthy feature of the table is the appearance in New York, and to a less extent in other states, of a course in general biology in the first year of the high school. This course first appeared in the high schools of New York City[4] as a more or less disconnected course in botany, zoölogy and human physiology. As it is now given in many of the schools it is a fairly unified course in general biology, treated from the physiological aspect.

(6) Another interesting development is the appearance of

[4]See minority report to Committee of Nine by J. E. Peabody, N. Y. State Dept. Bulletin 7, 1900, p. 775.

courses in elementary science in the first year of the high school. Such courses are scattered, no one state system of education being responsible for them. They appear to have arisen from a lack of proper preparatory work in the elementary school and aim frankly to introduce the pupil to the field of physical, chemical and biological science by the experimental method.

(7) An evident seeking after the practical in science is found in the courses in hygiene introduced in all years of the high schools of Boston and other Massachusetts schools and in the introduction of courses in elementary agriculture in some of the Western high schools.

The second part of the question, 1, *"Why do you believe this sequence of science to be the best order?"* brought out several interesting and rather startling facts.

The first is that in many schools at the present time the curriculum in science is planned without any sequence in view, the elective system allowing the pupil such free choice that he may elect almost any science in any year of his secondary school career. In other schools, on the contrary, but one science is allowed, this usually physics because of its place as a college entrance subject. This latter state of affairs has arisen because the American university has made such demands upon the secondary school that the curriculum in science is simply based upon the demand of the higher institution for physics, with overemphasis on the side of mathematics.[5]

A careful interpretation of answers received to the above question shows the factors governing the sequence of sciences in the secondary schools may be college entrance requirements, the syllabuses of state or city educational bodies, even the desire on the part of school authorities to offer inducements to the pupil in the form of elective subjects rather than a desire on the part of the educators for a pedagogic basis on which to build up courses in a sequence that will be best adapted to the needs of the child at the age which he or she is to receive them.

Schools offering biologic science early in the course with physics and chemistry later, after considerable mathematics, report almost uniformly satisfactory conditions. There is, however, considerable diversity of opinion as to the place of physiography. We may, for convenience, divide the schools into two groups, one giving physiography in the first year followed

[5]A revolt against such methods in physics teaching is seen in the report of secondary school teachers in physics recently, presented to and accepted by the College Entrance Board. See "Definition of Requirements in Elementary Physics," "School Science and Mathematics," 1909, pp. 572-579.

by biology in the second year and those giving biologic science in the first year followed by physiography. Some typical reasons given for taking physiography in the first year follow.

"In a way Physiography combines all the other sciences." "Able to handle larger classes better in Physiography." "Physiography acts as an introductory science, giving a broad view before specializing." "Physiography nearly related to the geography taught in the grades." "Increasing difficulty of mastery." "Increasing abstraction of phenomena and explanation." "In order of ability of student." "Regard to child's expression." "Of general interest." "Of interest to those who drop out early in the course." "As it was in the beginning, is now and promises to be indefinitely."

One incongruity results from this early teaching of physiography in states where the law requires human physiology to be taught in the first year of the high school. Here physiography and physiology are taught in different parts of the same year as totally unrelated subjects and without laboratory work in physiology. From ninety-four schools reporting courses in physiography in the first year, thirty-nine give entire or part year courses in physiography only while forty-two schools give physiology and physiography as unrelated courses. The remaining schools give physiography with some other science to fill out the year.

The following answers are taken at random from reports from schools giving biological courses in the first year:

"Best from standpoint of adaptation to needs of pupil." "Best adapted to ability of student." "Interest in introductory biology." "Biology least difficult for beginners; physiography requires a wider outlook." "Gives opportunity to build directly on the nature study of the grades." "Physiography last, based upon the other sciences." "Can be offered to more pupils and thus do more good." "Presents material of vital importance which all students should know when young, *i. e.*, at age of adolescence." "Presents fundamental principles underlying right living." "Most essential science first so that more will take it."

The second and third questions of the questionnaire have to do with the place of the biologic sciences in the school curriculum and the amount of time devoted to each subject. The answers to the questions were somewhat indefinite in many instances so that the material cannot well be tabulated. The following data are, however, of interest.

Of 501 courses reported in biologic science, 300 are year courses of from one to eight periods each, 201 are courses of half a year or less. These courses are classified as follows:

	YEAR COURSES.	COURSES OF HALF YEAR OR LESS.
Botany	111	74
Zoölogy	66	56
Human Physiology	58	70
Biology	60	1
Botany or Zoölogy or Human Physiology	5	0
	300	201

One hundred and twenty-two schools give year courses in biologic science (of from one to seven periods per week) in the first year of the high school out of two hundred and eighty-four schools reporting. Fifty-eight have half year courses in the first year. One hundred and twenty-nine schools give the equivalent of a year course in the second year and eleven schools half a year or less. In the later years of the high school the tendency is toward year courses in either botany or zoölogy. Half year courses in human physiology, coming later in the high school course, after physics and chemistry, have been introduced in relatively few schools, despite the recommendations of the Committee of Ten.

The number of school periods per week allotted to biology (and this is true of chemistry and physics so far as data are available) is in by far the greatest number of schools five periods per week of from 40 to 60 minutes each. A large number of schools especially in the Central, Rocky Mountain and Pacific divisions allow from six to eight periods per week. Frequently one or two of such periods is a double period used for laboratory work. Such periods, as in colleges, count as a single period of prepared work. In some schools, especially those giving courses in physiology only, the time allotment to biologic science is meager. A very few schools, notably in the South, devote but one or two periods a week to botany or zoölogy. In the courses in advanced human physiology the time allotted varies considerably, ranging from one period per week for a half year to five periods a week for a half year. For the most part biologic science seems to be recognized as a laboratory subject and time is given to pure inductive work in a laboratory with material in the hands of the pupil.

To the question, *"To what extent is the laboratory method used in human physiology?"* 82 out of 192 schools reporting devote two fifths of the time or more to laboratory or experi-

mental exercises, 71 devote a little time to such work and 39 give no experimental work. Illinois leads the black list with eleven schools reporting this latter condition while Massachusetts has eight and Pennsylvania five schools reporting text-book courses in human physiology. This condition is due almost entirely in Illinois and other states to state laws requiring the teaching of human physiology in the first year of the high school from text-books in the hands of the pupils. These laws, introduced and upheld by the Woman's Christian Temperance Union, promise to stand in spite of all the weight of opposition that scientific men can bring to bear against them. Bearing this fact in mind it seems wise to plan courses in biologic science for the first year of the secondary school and to make such courses physiologic in general trend rather than morphologic so that the *required* human physiology may have a scientific basis. This would do away with the incongruity of courses in physiology and physiography in the same year of the high school as unrelated subjects.

In answer to the question, *"Is the problem of the sex relation touched upon?"* 105 schools out of 212 reported that the subject was untouched; 80 schools taught sex relations in botany and zoölogy, allowing pupils to make their own inferences, while 28 schools treated the subject plainly in separate classes or by a series of lectures given by physicians or qualified persons outside the faculty. In view of the immense significance of sex physiology and the growing interest in the proper presentation of this subject the above figures are of interest.

Some of the most interesting data were compiled from the answers to the following question: *"On what phases of biological subject matter do you place the most emphasis; morphology, physiology, natural history, taxonomy, relations to man, human biology? Why?"*

Investigators in biologic research know the trend of such work during the past decade. From problems of pure morphology, cell lineage and the like the research student has turned to problems of physico-chemical nature or to the realm of pure physiology. It would seem that the secondary school reflects to a remarkable degree the aspect taken up by the college and university student.

In reading over the answers one notices the swing of the pendulum in secondary school toward the physiological aspect of biology teaching. In tabulating the results from this answer it becomes necessary to remember that the emphasis on the

various aspects of the work are often inseparably connected. Morphology in the modern secondary school laboratory is the frame on which the other more vital truths of the biological sciences are hung. The figures which follow are only of value when we bear in mind that emphasis may be placed by the same teacher

	PHASE OF BIOLOGIC SUBJECT MATTER EMPHASIZED						
	Morphology	Physiology	Natural History	Taxonomy	Ecology	Relations to Man	Human Biology
New Hampshire	2	2	2	1	2		
Massachusetts	19	25	11	6	21	21	6
Rhode Island	2	2			2	2	
Connecticut	6	3	2	1	4	5	1
New England States, Total	29	32	15	8	29	28	7
New York	16	25	9	1	18	23	13
New Jersey	3	4	4		3	3	1
Pennsylvania	9	12	7	1	7	8	3
Maryland						1	
District of Columbia	4	3	2	1	3	2	1
Middle States, Total	32	44	22	3	31	37	18
Virginia	1	1	1			1	
Kentucky	2	5			3	3	1
Tennessee		1	1				1
Louisiana	1	2	1				
Oklahoma	1						
Southern States, Total	5	9	3	0	3	4	2
Ohio	7	16	9	3	12	14	5
Indiana	8	11	2	1	9	6	2
Illinois	15	17	5	2	15	18	9
Michigan	9	11	4	1	7	5	
Minnesota	4	4	1	2	2	2	2
Missouri	7	7	2	1	4	4	2
Iowa	3	6	3	1	5	3	2
Wisconsin	10	9	6	3	10	10	6
North Central States, Total	63	81	32	14	64	62	26
Kansas	2	1	1		2	1	
North Dakota	1	2	1		2	1	1
Nebraska	1	2	1		1	1	
Colorado	2	2		1	1	1	
Utah	1	1	1				
Rocky Mountain States, Total	7	8	4	1	6	4	1
California	1	2			1	2	1
Washington	2	2		1	2	1	
Pacific States, Total	3	4		1	3	3	1
GRAND TOTAL	139	178	76	27	136	138	55

on several of the various aspects of the science. We can then read a composite picture from the parts of such data.

(To be continued.)

THE USE OF THE MICROSCOPE IN BOTANY.

Teachers of botany appear to find it difficult to decide just how much time to devote to work with the microscope. Some very excellent teachers contend that this instrument has no place at all in high school botany, while others make free and constant use of it. Having in mind this difference of opinion, the writer recently endeavored to ascertain the opinions of the pupils themselves on the subject, by asking his classes the following questions: Did you dislike or enjoy learning to use the microscope? Do you dislike or enjoy using it now? In your botanical work would you prefer to substitute other work for that requiring the use of the microscope? Fifty-nine pupils were questioned. About two thirds of these had just completed their first quarter's work in botany and the rest were in their second half year and were just emerging from a pretty extended microscopic study of the algae and fungi. The result was a trifle disappointing, for it reads rather too smooth to seem true, but every one of the fifty-nine asserted that they enjoyed learning to use the microscope, that they like to use it now and would rather begin botany with a microscopic study of cells than with a study of seeds using a simple lens. It may be said that in the school in question, a year is devoted to botany and the beginning student is plunged at once into the intricacies of nuclei, chromoplasts, chloroplasts, chromosomes, cell-divisions, starch grains, the circulation of protoplasm, plasmolysis, osmosis and allied subjects. Of course there are enough microscopes for all the pupils, though the fact that several pupils may have to use one microscope ought not to affect the result. Those teachers who have never begun a class in this way are likely to criticise this method as being too difficult for the beginner, apparently overlooking the fact that the student must get familiar with the structures found in cells sooner or later and might as well begin at the beginning. The writer is inclined to defend the method because, first, as we have seen, the student likes to use the microscope and in so doing manages to work up enough interest in botany to carry him well into the thick of the course, and, second, because he at once lays down a foundation knowledge of cells that will enable him to study intelligently such minute structures as root-hairs, palisade cells, stomata and guard cells and the parts of the vascular bundle as he comes to

them in the course, and which will give him the ability to understand such processes as photosynthesis, respiration, transpiration and digestion. It is hard to conceive how a pupil who has merely read about chloroplasts in a book can form much of an idea of carbon assimilation from such sources. He may be able to recite all about it, but that does not argue that he understands it. In beginning with the microscope, the teacher devotes some time to "showing off" the instrument by the use of various attractive mounts. Some of the things that have kept the attention of the pupil concentrated upon the work are diatoms, pleurococcus and other algae, starch grains, cells of a potato or apple, blood corpuscles, various infusoria, scales from a butterfly's wing, etc. A visitor going into this laboratory and finding the pupils interested in the antics of the paramœcium might insist that the study was not botany, but that only shows how very little he knows about it. C. N. W.

BALANCED SEA-WATER AQUARIA.

By L. B. Spencer,
New York Aquarium.

Editorial Note.—For several years the New York Aquarium has kept many high schools and some elementary schools of New York City supplied with balanced sea-water aquaria. We have prevailed on Mr. Spencer, who has had charge of this work at the aquarium, to tell the readers of School Science and Mathematics what a school aquarium may contain and how it should be cared for. If any of our readers have had experience in shipping live sea animals inland, we should be glad to hear from them.

A five gallon round glass aquarium is the kind and size used in the schools of all the boroughs of New York City. The aquarium jar should be placed in a window, or so near one that it will get strong light. If the sunlight strikes it, place a screen between it and the window. Do not pull down the shade, for the aquarium requires actinic light, but not direct sunlight, unless the water becomes cloudy. If that happens direct sunlight for a short time will help it. Cover the bottom with a layer of coarse, clean beach gravel, place on it a flat piece of rock about three by four inches square and about one inch thick on the room side of the aquarium, one inch from the glass. Put in the sea water, filling the jar up to the bend on the top edge. Place two brown sea anemones (Metridium marginatum) or five or six of

the Sagartia genus on the piece of stone. Put in five or six fairy shrimps (Palaemonetes vulgaris); three to five sheepshead lebia, (Cyprinodon varigatus), a beautiful little aquarium fish. If these are not to be had, use the more common salt water killie fish (Fundulus heteroclitus). The snail (Nassa obsoleta) is interesting to watch, and is a scavenger. The hard clam (Venus mercenaria), the soft clam (Mya arenaria), the oyster (Ostrea virginica) can be used, if their valves are open, it is well to touch them with the feeding stick once a day. If they are all right, the valves will be closed at once; if not, take the sick or dead one out promptly, or the water will be spoiled by the decomposition of the mollusk. The acorn barnacle (Balanus balanoides) usually does well for a time. The sea lettuce (Ulva latissima) is the only practical plant to supply oxygen for a marine aquarium. Use pieces of cork about one inch square by one quarter of an inch thick, under each sheet of Ulva to keep it at the top of the aquarium. Use five or six sheets of the lettuce ten or twelve inches in diameter. Do not let the Ulva sink to the bottom. Once a week make up for evaporation by pouring fresh water into the aquarium.

Feed the animal life three times a week, with very small pieces of the soft part of the hard clam or oyster. The fishes and shrimps will take the pieces as they float towards the bottom. Put a small piece of the food on the end of the feeding stick, and place it among the tentacles of the anemones, as they cannot reach out for dead food. Be extremely careful not to put in more food than the animals will eat. If any pieces of food are not eaten they must be removed a few hours after feeding to prevent trouble from decay. For this purpose use a glass tube with a half inch hole, length about eighteen inches. Place the thumb tightly on top of the tube, put the open end into the water to the bottom among the pebbles, raise the thumb slightly to let the air escape, and the water and pieces of food will instantly rise in the tube. Press the thumb at once tightly on top of the tube, raise it out of the aquarium, hold the lower end in a small scrim net, held with the left hand over the aquarium. Raise the thumb and the water will run out. The net will retain any solid substance. If proper care is exercised the balance can be maintained for years. A balanced aquarium in the laboratory of the New York Aquarium was filled with sea water and stocked by the writer fifteen years ago last June, and the water has not been changed.

If from any cause the water becomes cloudy, take a tin or glass vessel, dip up some of the water, hold the vessel eight or ten inches above the aquarium, and pour the water back. The stream of water catches up oxygen and carries it into the aquarium. Repeat this operation two or three times a day until the water clears up. Once or twice a year it will probably be best to clean the gravel. In doing this, remove the fishes and shrimps with the small net, then carefully siphon out the water, within about an inch of the bottom, remove the Ulva, take out all of the animal life, which with the fishes and shrimp can be put into the water taken from the aquarium. Then strain the inch of water, wash the gravel and the inside of aquarium; put back gravel, water, animal life and Ulva, and the aquarium should be all right, with proper care, for many months.

THE CULTURE AIM IN PHYSICS TEACHING.[1]

By Frank B. Spaulding,

Boys' High School, Brooklyn.

No two persons mean precisely the same thing by culture. A scholarly recluse and a leader of polite society, after a vain attempt at conversation, may be expected to vote each other wholly devoid of culture. In fact, each represents a high degree of cultivation in a particular form of intelligence. Matthew Arnold, in the preface to "Literature and Dogma," defines culture as "the acquainting ourselves with the best that has been known and said in the world, and thus with the history of the human spirit." But the human spirit is not always the literary and speculative spirit of Matthew Arnold. Let us take a definition more modern in tone furnished by W. H. Brooks, who says, "Culture consists of acquaintance with all the old and new results of intellectual activity in all departments of knowledge, so far as they conduce to welfare, to correct living and to rational conduct." Physics is a department of knowledge which can offer to the student results of both old and new intellectual activity conducing in a very direct and important way to human welfare and rational conduct. However small a part such knowledge may have played in the development of ancient and mediæval minds, no culture is entitled to be called

[1]Read before the joint meeting of the Physics Club of New York and the New England Association of Physics Teachers, at Hartford, Conn., April 24th, 1909.

general or *liberal* in this generation which does not include some appreciation of the significance of the physical phenomena which bear so directly upon our daily lives.

Having chosen a definition sufficiently inclusive, let us make it more definite by exclusion.

President Stryker of Hamilton College recently said to the boys of our school: "Do not come to Hamilton if you wish to get the degree of A.B. for a course in plumbing and pastel crayon." Now I do not agree with Dr. Stryker that the only true culture is classical culture, but I do agree that an important distinction is to be made between culture and utility as the aim of education. It is not always an easy distinction to make. Only in very rare instances does either aim appear singly. In the trade school, the commercial school and the technical school, utility is generally the predominant aim; general culture is sacrificed to special training. The pupil masters his block plane or his ledger or his formulæ of mechanics, not for the sake of insight into life, but to increase his efficiency as a bread winner.

I would by no means belittle the importance of this special training; it is becoming more and more necessary as specialization in every department of life increases. But I would defer it until the pupil has attained the largest all round growth which his financial circumstances and his intellectual capacity permit. "What science and what practical life need," says Dr. Nicholas Murray Butler, "is not narrow men but broad men sharpened to a point."

The college preparatory idea gives rise to a peculiar form of utility aim. The preamble to a list of laboratory exercises issued by the Physics Club of New York ten years ago, suggests among other aims, "power in manipulation of apparatus and in overcoming the difficulties of scientific investigation." The idea was that the physics teacher should give his pupils mastery of the tools with which they might do the advanced work of the higher institutions, just as the Latin teacher should send out pupils well grounded in declension, conjugation and rules of syntax. This idea has so little vogue with physics teachers to-day that it is not worth discussion. We have generally concluded to leave the training of scientific investigators to the universities; but, while we no longer aim at preparation for college work, I fear many pupils and perhaps a few teachers

aim at preparation for college entrance examinations. Could any utilitarian aim rank lower?

Again, the culture aim is to be distinguished from that of *mental discipline.* There are still some people who believe that the mental anatomy consists of a memory muscle, a reason muscle and a judgment muscle, etc., which need only to be exercised on loads of gradually increasing difficulty to bring them up to any desired power. But the bank cashier who never forgets a signature frequently forgets his umbrella. We have all met those who reason well in mathematics and badly in politics, or whose judgment is excellent in music and poor in morals. The truth is, the mental functions are performed with ease and precision in relation to any subject in proportion as knowledge of that subject is full, definite and well organized. It is no effort for the botanist to remember the appearance of a new plant; the electrical engineer understands a new type of generator at sight. The Great Teacher enunciated a broad philosophical principle when he said, "To him that hath shall be given and he shall have more abundantly."

In a general secondary school course, the chief aim of physics teaching is not commercial utility, not preparation for advanced study, not mental discipline. It is to acquaint the pupil with the great discoveries of great minds past and present, to enable him to intelligently adapt himself to this twentieth century world in which the application of physical principles play so important a part, and above all to cultivate a scientific attitude of mind, a mind which looks upon the universe as rational, not arbitrary; as dynamic, not static.

And now, lest I seem to leave this discussion in the thin air of generalities, let me illustrate the foregoing distinctions by a number of problems and exercises characteristic of these various aims:

1. At what distance below the earth's surface would one pound weight weigh only an ounce?

2. Compute the strength of a magnet pole which repels a similar and equal pole with a force of 100 dynes at a distance of 3 cm.

3. What acceleration will a force of 8 dynes impart to a mass of 20 grams?

4. Find the resultant of two forces of eight and ten pounds respectively acting at an angle of 60 degrees with each other.

5. Find the centrifugal force of a mass of 12 grams moving with a velocity of 20 cm. per second in a circle of 10 cm. radius.

I hardly need say that the aim, conscious or subconscious, of a teacher who assigns such problems is either utility or mental discipline. They are easy. The stupidest pupil could solve them if he remembers a formula or a process. They involve no physics, merely mathematical abstraction. Thousands of pupils have acquired facility in the solution of such problems and passed physics—perhaps I should say, "passed by physics"—without ever once perceiving a relation between the problems and any physical phenomena. In fact the first has no relation to human experience. The second connects with facts which come within the observation of only one person in a million. The others could be made concrete and made to throw light on the common experience of the pupils, for instance:

1. A motor car weighing 1,800 pounds is moving at the rate of 30 miles per hour. If the brakes can stop it in a space of 40 feet, what frictional force must be developed between the tires and the road?

2. Construct the angle at which a railroad curve of 400 feet radius should be banked to accommodate a speed of 20 miles per hour.

Difficult? Certainly. That is why so many stop with the theoretical and formal; it is so much simpler than the real. But if your pupils have not the time or the ability to attain to at least something of this concrete character, what valid excuse is there for introducing the mathematical treatment of acceleration, force, and curvilinear motion? Is there not enough *real* physics within their grasp?

Time forbids further illustration. Let me close by saying that, however fine a notebook the pupil may produce, however glibly he may recite definitions and laws, with whatever facility he may juggle formulas and construct optical diagrams, the study of physics has come far short of its mission if it has failed to make abundant addition to the meaning of the phenomena which meet his senses on every hand.

THE PURPOSE AND ORGANIZATION OF THE TEACHING OF CHEMISTRY IN THE SECONDARY SCHOOLS.

(Symposium continued from November, 1909, issue.)

IX. By W. Segerblom.

Phillips Exeter Academy, Exeter, N. H.

If chemists knew the one principle underlying *all* chemical action under *all* conditions they might safely start with that principle and deduce therefrom the knowledge obtainable at the laboratory desk. In that case chemistry would become a deductive science and much of our present educational paraphernalia, laboratories included, would be worthless. But, as we well know, we collect our facts, arrange and study them, and formulate statements which we designate "general principles." Then we are tempted to congratulate ourselves on the splendid results of our inductive endeavors. Are we consistent if we collect our "general principles" into one chapter and present that to our students at the *beginning* of the school year? Are we fair to the students if we do not let them gain a sufficient stock of facts and a sufficient acquaintance with chemical substances and their reactions before we present certain general principles of which we have ourselves gained an intelligent conception only after considerable laboratory work? Are the pupils not in danger of accepting the "general principles" which we know hold absolutely only for cases actually studied as the one principle underlying all chemical action? For most students individual work at the desk in a chemical laboratory is in itself such a novel experience and so full of surprises that we may well wait till they have become accustomed to the new experiences. When the ordinary student mind feels the need of theory to help set in order his rapidly accumulating facts, then he will reach out naturally for it and will assimilate it with less effort. With this point of view constantly guiding us, it should be an easy task to lay out a consistent and logical first year course in chemistry.

To my mind such a course should begin with solids instead of gases, for it is natural that the beginner should be able to get more tangible results from the more tangible substances. Such metals as copper, zinc and magnesium and such non-metallic solids as sulphur and phosphorus lend themselves admirably to this plan. The ease with which these substances oxidize leads

directly to oxidation, oxides as a class, and the formation and composition of certain acids; with the next logical topic, the action of acids on metals, we are fairly under way in our experimental study of chemical action. Work on gases should be introduced only as soon as the students have learned to handle the solids intelligently from the chemical point of view. In all this work it is of the utmost importance that we do not turn the students loose in the laboratory and expect them to invent their own methods of working; it is as essential to teach them *how to work* as it is simply *to work*. If secondary school-teachers of chemistry should persistently teach a *few* things well and not try to touch the whole field of chemical science, perhaps their friends —the college professors of chemistry—would gladly accept the fruits of these labors and might find a more solid foundation on which to build their chemical superstructure.

If at present the burden falls unduly heavily upon the secondary school-teachers of chemistry that should in no wise discourage them for it simply increases their opportunity for achieving much wished for results. One of the objects to be constantly striven for is to turn out students who are not chemical investigators but who are able to think their own way out of difficulties of reasonable severity. To attain this the instructor should have and constantly hold to a proper sense of proportion of the various topics he takes up with the class. It is well to remember that students in the laboratory often learn as much from what the instructor does not say as from what he does say, It behooves him, therefore, to maintain always a scientific attitude of mind and to reflect those traits of mind, such as unprejudiced honesty of observation, that he expects his students to manifest, and the most effective time to do this is when he is working out the unexpected problem at the student's elbow.

Originality of thought on the student's part should of course be encouraged. How much more of real value was contained in the conclusion that a certain student reached through his own thinking that vacuum was "drawn out space" than was the labor of another student who spent nearly an hour carefully preparing a flask for determining the specific gravity of carbon dioxide and then when he was ready to replace the air in the flask with carbon dioxide did not remove the rubber stopper from the flask, with the result that the pressure from the gas bag shot the flask across the desk and smashed it against the locker across the next aisle.

Chemical problems should test a knowledge of chemistry rather than arithmetical ability; for that reason the figures might well be selected so as to reduce the actual mathematical calculation to a minimum. I find in my own work that it often helps the student if I indicate how many decimal places should be saved throughout the work if decimals appear.

So much has been said on the subject of equation writing that I hardly dare to add another word, but I do wish to emphasize the necessity of using equations only as a mode of expressing weightier facts instead of as things by themselves. It follows, then, that equations written by the students should so far as possible be based on experiments actually performed.

My own experience has convinced me that the formal lecture in chemistry has little place in a first year course. It *may* be used to advantage with the second year class, but the first year should not be wasted in lectures when there is such a splendid opportunity for pupils to gain first-hand knowledge personally at the laboratory desk.

Valuable as is the recitation or the quiz I have found that the casual question while the student is busy with his experiments is often more fruitful in productive thinking.

Concerning the directorial organization of the teacher's work it should not be necessary to speak in detail in these days of highly organized business management, but since secondary school-teachers presumably have not had much training in business management it might be to their advantage to subscribe for and read some good periodical on business methods. Many of the labor saving devices improvised by successful business men may be adapted to the needs of the secondary school-teacher. A judicious use of the card system suggests itself first. As another instance, ordering apparatus by multiples of ten instead of by the dozen enormously economizes the bookkeeping and stock room work.

I cannot understand the mental attitude that says chemistry should be "made attractive" to the students. Chemistry is, in and of itself, so intensely attractive, that it seems to me our only danger is in making it unattractive by dressing it up in inconsistent, illogical and unscientific methods of presentation.

The secondary school teacher of chemistry has one advantage over his fellow laborer—the college teacher—in that he has better opportunity for research of an extremely profitable kind.

The investigation of compounds and processes he may well leave to the college man with his larger equipment and command of research assistants. The secondary school teacher can research in fields that are closed to the college man, that is, he can investigate, devise, try, improve and finally settle the best methods of producing satisfactory studential products from first year classes in his subject. Surely the time is ripe for more productive elementary chemistry teachers.

Finally the teacher of chemistry should both aspire and inspire—aspire, himself, to reach perfection in his calling, and inspire his students to ever higher effort. If he does the first, the second will take care of itself.

TWO EXPERIMENTS RELATING TO CHANGE OF STATE.

By Elmer E. Hall,

University of California.

So far as the writer is aware, the only experiment in heat of fusion, in use in elementary courses, is one determining the heat of fusion of ice. Also, the heat of vaporization of water at its boiling point is the stock experiment to show the changes in internal energy in passing from the liquid to the vapor state. From the latter experiment the student often draws the inference that heat of vaporization applies only to a boiling liquid. In order to give students in the freshman course further insight into the important changes in internal energy accompanying change of state, without repeating work done in the high school, the writer has used two experiments outlined below.

In the exercise on heat of fusion, Wood's alloy is used. The composition of this alloy by weight, as shown in the Smithsonian Physical Tables, is lead 25.85, bismuth 52.43, tin 14.73, and cadmium 6.99 parts. The melting point as given in the same book of tables is 75.5° C. The method of mixtures is employed, the melted alloy being dropped in a calorimeter containing cold water. A known mass of the alloy is placed in a nickel crucible, the mass and specific heat of the crucible being known. The crucible and contents are heated in a water bath, slowly after the temperature of 60° is reached, and the melting point is determined. The thermometer should be placed in the water

bath and not in the metal, as the liquid metal wets glass and hence the thermometer cannot be removed without changing the mass of the metal used. The thermometer cannot safely be left in the alloy since the metal contracts on solidifying, thus endangering the thermometer. If the thermometer is left in the metal, and stands the strain, it can later be removed, since the alloy shows anomalous dilation at the fusion point. After the melting point of the alloy has been found, the water in the bath is brought to the boiling point, care being taken that no water gets inside the crucible. The crucible is then removed, quickly wiped on the outside, and carefully dropped into the calorimeter, right side up, letting the water run into the crucible, thus more quickly cooling the metal. The ordinary calorimeter observations are made, and corrections made for losses by radiation. The specific heats of the liquid and the solid alloy may be given, or they may be determined by a separate experiment. When cold the alloy can be dumped out of the crucible leaving the crucible clean.

This experiment was worked by nearly 300 students in the freshman class at the University of California during the past term, and gave excellent results from the standpoint of workableness, new ideas involved, interest on the part of the students, and uniform quantitative results. For the first 25 papers, good, bad, and indifferent, which came in, the average melting point was 76° C. and the average heat of fusion was 8.2 calories per gram per degree centigrade. The maximum value was 8.9 and the minimum value was 7.3. A careful student made three separate determinations, obtaining 8.40, 8.40, and 8.52. The experiment affords an excellent opportunity for a discussion of the relative accuracy with which the different measurements are to be made. The ordinary commercial metals are used, and if desired a variation in the composition can be made. For example, the alloy containing 4 parts of bismuth, 4 of lead, 1 of tin, and 1 of cadmium has a melting point of 85° C. If 4 parts of bismuth, 2 of lead, and 1 of tin be used, the melting point is 96° C. Eight parts of bismuth, 4 of lead, 2 of tin, and 1 of cadmium give a good working combination with a melting point slightly lower than that for the combination given in the Smithsonian Tables.

In nature, vaporization usually takes place at atmospheric temperature rather than at the boiling point. Hence an exer-

cise was chosen on the heat of vaporization at room temperature. Dry air is made to bubble through the liquid used, thus increasing the free surface and producing rapid evaporation. The loss in weight of the liquid gives the amount evaporated, while from the fall of temperature of the liquid and calorimeter, together with their masses and specific heats, the heat loss can be determined and the heat of vaporization calculated. The liquid to be used is placed in a calorimeter cup, the initial temperature being as far above as the final temperature is below room temperature, thus losses by radiation are compensated. A small foot bellows may be used to force the air first through lime or calcium chloride and then through sulphuric acid to dry it, the dry air being allowed to bubble up through the liquid in the calorimeter. Or, the air may be drawn through the apparatus by an aspirator. Using for calorimeter one glass beaker inside another glass beaker, three consecutive values obtained for heat of vaporization of ethyl alcohol at room temperature were 211, 219, 209. Water does not give as uniform results since the amount evaporated is less. The following values, however, are representative, 580, 611, 567, the correct value at the room temperature being about 587. The results obtained by an elementary class by this method are as consistent as those usually obtained by the ordinary method for determining heat of vaporization at the boiling point, and the two experiments together bring out clearly that the heat of vaporization is a function of the temperature. The method is of course applicable to finding the heat of vaporization at temperatures other than room temperature, but errors due to radiation losses become more troublesome. While these experiments were originally designed for freshmen students they have proved so simple and workable that they may be found useful as alternative experiments in connection with high school work.

ARTIFICIAL RIPENING OF FRUITS.

Mr. A. E. Vinson of the Arizona Agricultural Experiment Station, in *Science* of October 29, reports successful attempts in ripening fruits by use of chemicals. By subjecting sprays of unripe dates for 12 to 15 hours to the vapor of acetic acid, the dates soon ripened. The flavor is said to be quite as good and ripening more even than when normal processes are used. It is thought that dates may soon be handled as bananas are now, by shipping them green and ripening them at their destination when needed. Since green dates as green bananas ship better than ripe ones a distinct advantage is had. O. W. C.

A SIMPLE FORM OF FREQUENCY METER.

By J. M. Kuehne,
University of Texas, Austin.

In the prosecution of a research involving the use of an alternating magnetic field the writer had occasion to make determinations of the absolute frequency of the alternating current used. No reliable frequency meter being at once available, an instrument was improvised which proved so convenient and easy to set up, and withal of such satisfactory accuracy, that it was believed a description of it might prove of interest to other readers of School Science and Mathematics.

The method consists simply in sending the current through the wire of a sonometer, which wire is made to pass between the poles of a horseshoe magnet, and then altering the length of the vibrating segment by means of the ordinary sliding bridge until the wire vibrates without showing beats. Beats as slow as one in 8 or 10 seconds are easily observable, which means that the adjustment to synchronism can be made to within about one tenth vibration per second. All that remains is to determine the length of the vibrating segment required to give some known frequency, such as that of a tuning fork. The frequency of the current is then at once given by the formula

$$NL=N'L'.$$

Ordinarily it will be considered that this tuning of a sonometer wire to a fork is not an extremely accurate procedure; but by a simple device which has for some years been in use in our elementary laboratory an accuracy of 1/4 or 1/5 vibration per second is easily attainable, without in any way relying on the observer's sense of musical pitch. This device consists in using a very light bridge on which there is a place to press the handle of the tuning fork, and observing the vibration of the wire automatically set up by the fork, whenever the length of the wire is such as to be very nearly in tune with the fork. If the fork is a good strong one, the vibration of the wire will be vigorous and sustained enough to make the observation of beats an easy matter. A piece of white paper for a background will facilitate matters, by making the vibrations more easily visible.

If an electrically maintained tuning fork of known frequency is available the wire may be still more easily and accurately tuned to it, by merely sending the intermittent current which is ener-

gizing the fork, through the sonometer wire, a magnet being placed over the wire as in the case of the alternating current.

In order that the application of the formula $NL=N'L'$ shall not involve a serious error it is evident that the conditions under which the formula holds should be as completely as possible fulfilled. That is, the wire must be uniform, and the tension constant, and, since the formula really applies only to flexible cords, the wire should be long and comparatively thin. It is of course best to use a fork whose frequency is not too different from that of the current to be measured, but results reliable to considerably less than 1% can be obtained even if the ratio of frequencies is as great as 3:1.

In the apparatus as built up for the particular purpose in hand, a No. 23 brass wire was used, stretched by a 2 kg. weight. Tuned to a fork of frequency 128, L was found to be 39.4 cm. A weak horseshoe magnet was placed with its poles on opposite sides of the wire, near the middle of the vibrating segment. The alternating current (about 2 amperes) was passed through the wire, and a length of 85.2 cm. was found to give resonance without noticeable beats. Hence, the frequency of the current was

$$N=\frac{128\times 39.4}{85.2}=59.2 \text{ cycles per second.}$$

A change in frequency sufficient to require a change in length of 1 mm. was easily discernible. This indicates a change in frequency of less than one tenth vibration per second. The apparatus may easily be modified for special purposes. For example, a thinner wire (No. 27 brass wire) and smaller tension (about 900 g.), together with a stronger magnet were used. A large resistance, 1,500 ohms, was placed in series with the wire and about 75 volts applied. Although the current was now only .05 amperes, the vibrations were amply large to be observed, and tuning could be very easily and accurately effected. In this case, with a tuning fork of frequency 128, L was found 41.8 cm. Tuned with the alternating current, resonance was obtained with L' equal to 90.6 cm. Hence N' equal 59.0 cycles per second. In this form the instrument may be connected like a voltmeter; in fact, the sonometer wire may be placed in series with the voltmeter, the latter supplying the resistance necessary to reduce the current to negligible values, while the small resistance of the sonometer wire will have no appreciable effect on the voltmeter readings.

A THERMOSCOPE AND ITS USES.

By E. J. Rendtorff,

Lake Forest Academy, Lake Forest, Ill.

(*Continued from December issue.*)

Absorption.

16. Heating Through the Absorption of Gases by Solids.

Cover (2) with a large inverted beaker full of ammonia gas and note the indicator reading when equilibrium is produced. Cover the top of the bulb with powdered charcoal that has previously been heated and allowed to cool. Note the slight increase of temperature.

17. Absorption of Gases by Liquids.

Partially fill (3) with water and place an inverted beaker of ammonia gas over it.

18. Cooling by Diffusion of Absorbed Gases.

Place strong ammonia water in (3) and cork. When the indicator comes to rest remove the cork and note the cooling produced by the diffusion of the ammonia.

19. Absorption of Moisture by Wool.

Connect (2) and place over it a large beaker filled with water vapor. Adjust the indicator and place some dry wool, previously dried in a desiccator, on the flat surface of the bulb. A slight heating results, due to the condensation of the water vapor.

20. Hygroscopic Power of H_2SO_4.

Repeat Experiment 19, using a little concentrated H_2SO_4 instead of the wool.

21. Heating Produced by Taking up Water of Crystallization.

Connect (3) and partially fill with water. Add some powdered $CuSO_4$ from which the water of crystallization has previously been removed by heating.

Conductivity.

22. Conductivity of Metals.

Take a bar of metal and bend about 7 cm. of one end at right angles. Push the bent end through a cork and insert in (3). Heat the rod about 10 cm. from the thermoscope.

To illustrate the different conductivities of various metals, bend another bar of the same dimensions and apply to the other branch of the thermoscope. Cross the bars at equal distances from the thermoscope and hold in a Bunsen flame.

23. Good and Bad Conduction of Heat.

Attach both (2). Place a piece of woolen cloth over the flat face of one bulb and a thin sheet of metal over the other. Place weights of equal temperature and mass on both wool and metal. Note the different rates of heating.

24. Conductivity Depends on Molecular Structure.

Repeat Ex. 23, placing over (2) thin pieces of the same kind of wood of equal thickness, but with the grain running in different directions.

25. Conductivity of Liquids.

Break the bottom of a bottle and introduce (1) from its neck. Cover the bulb about 5 mm. with water and wait until equilibrium is produced. Add a little ether and ignite. No change of the indicator results.

26. Conductivity of Gases.

Connect the inner bulb of (5) to the thermoscope and stopper up the opening of the outer one. Immerse in hot water. Note the elevations of the indicator after a specified time.

27. Effect of the Density of a Gas on its Conductivity.

Repeat Experiment 26, having first partially exhausted the outer bulb.

28. Principle of the Dewar Bulb and the "Thermo-bottle."

Completely exhaust the air in the outer bulb of (5) and repeat Ex. 26. The inner bulb is now heated by radiation only.

29. Function of Bad Conductors in Clothes Material.

Place both (4) in bottles containing the same volume of equally hot water. Cover one bottle with flannel or woolen cloth and note the retarded cooling. Also repeat Experiment 23, using pieces of woolen and cotton cloth.

Change of State.

A. Solids to Liquids and Vice Versa.

30. A Mixture of Ice and Water Remains at a Constant Temperature Until All the Ice Is Melted.

Partially fill (3) with cold water and add crushed ice. The indicator falls rapidly to a certain level and then remains constant until the ice is all melted.

31. Freezing Mixture.

Fill (3) with chopped ice. Note the constant indicator reading and then add salt. Greater cooling results at once.

32. Amorphous Solids Do not Have a Definite Melting Point.

Attach (4) and immerse in a beaker of melted paraffin. A constant cooling ensues while the paraffin solidifies.

33. Depression of the Freezing Point of Solutions.

Place (1) in melting snow or ice and adjust the indicator to zero. Now immerse the bulb in a beaker containing a solution of some salt and surround the beaker with a freezing mixture of salt and ice. When the ice forms on the surface of the solution again read the indicator and note the depression of the freezing point.

34. Solutions of Electrolytes of the Same Molecular Concentration Have the Same Freezing Point.

Make two solutions of some soluble salts by adding to 200 cc. of water amounts of these salts proportional to their molecular weights. Repeat experiment 33 for each solution. The depression of the freezing points will be practically the same for both solutions.

The law holds strictly only for dilute solutions, as the extent of dissociation in strong solutions varies with different salts. It also holds only when the electrolyte dissociates into the same number of ions.

35. The Depression of the Freezing Point of Solutions of the Same Molecular Concentrations Is Twice as Large for Electrolytes as for Non-electrolytes.

Make a sugar solution of twice the molecular concentration of those used in the previous experiment. Repeat experiment 32. The depression is practically the same as for the electrolyte, which dissociates into two ions, while the non-electrolytes do not. This law holds strictly only for dilute solutions, and in cases where the electrolyte dissociates into two ions.

36. Expansion of Water on Freezing.

Fill an air thermometer bulb and half the stem with ice water. Connect it to the thermoscope and place the bulb in a freezing mixture. Note the elevation of the indicator produced by the expansion of the ice.

37. Heat of Fusion of Ice.

In each (3) place 15 cc. of water at about 20° C. Place about 2 g. of ice in one bulb and the same weight of ice water to the other. The ice produces much the greater cooling.

B. Liquids to Gases and Vice Versa.

38. Cold by Evaporation.

Connect (2) and moisten with water or some other volatile liquid.

39. Effect of Air Currents on Evaporation.

Pour water on the flat surface of (2) and note the indicator reading. Hold a piece of cardboard on the side of the bulb and blow over it, so as not to strike the bulb itself.

40. Effect of Saturated Air on Evaporation.

Connect (3) and partially fill with ether. Place the bulb under a bell jar. The cooling soon diminishes. Remove the jar and note the increased cooling.

41. Effect of a Diminution of Pressure on the rate of Evaporation.

Attach (3) partially filled with water and note the indicator reading. Connect the bulb to an air pump and exhaust. More rapid cooling results.

42. The Hygrometer.

Attach both (1) and note the indicator readings. Partially cover one bulb with a piece of flannel and let it touch a beaker of water. The indicator lowers, due to the evaporation of the water.

43. Influence of the Dryness of the Air on Evaporation.

Attach (3) partially filled with water and note the indicator reading. Place the bulb in a desiccator containing concentrated H_2SO_4, or dry $CaCl_2$. The cooling is increased.

44. Method of Cooling Drinking Water by Evaporation.

Connect (1) and place in a porous earthenware vessel filled with water at the room temperature. The evaporation of the water on the outside of the vessel lowers the indicator.

45. Heat of Vaporization of Water.

Connect both (3) and add 15 cc. of water. Place 1 cc. of water in a tt and connect a bent tube reaching into the water of one bulb. Boil the water and let the steam condense in the bulb. Add the same volume of boiling water to the other bulb. The steam produces much the higher elevation of the indicator.

By adding the boiling water to the second bulb until equilibrium is obtained a rough value of the heat of vaporization can be determined.

46. Elevation of the Boiling Point of Solutions.

Place (1) in boiling water and adjust the indicator to zero. Now immerse the bulb in a boiling solution of some salt and note the elevation of the boiling point.

(*To be continued.*)

PLAIN CROSS-SECTION DRAWINGS ON CARDBOARD FOR SCHOOL USE.

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In physics and chemistry as in other subjects there are certain fundamental principles to be fixed in the minds of our pupils if any real understanding of either subject is to be obtained. It is the purpose of this paper to discuss briefly the importance of drawings in inculcating these scientific principles, and to urge that sectional drawings be more extensively used in text-books and by teachers and students. While the discussion will be of interest to all science teachers, it is my hope that it will be of special assistance to those who have but recently entered upon the work of teaching.

That more attention should be given the matter of drawings on the part of our teachers and text-book writers needs little argument. Our text-books and laboratory manuals are in general much better illustrated to-day than were those in use even ten years ago. The cuts employed to represent various pieces of apparatus are much clearer and greater pains is taken to illustrate the actual commercial form of the apparatus under consideration. In some of the more recent texts we may find excellent examples of color work, especially in connection with the subject of light. In the matter of sectional drawings, however, most texts are still deficient. The drawings shown in many books even for simple pieces like the ordinary force pump, for example, closely resemble patent office drawings, while the drawings to illustrate the arc lamp with automatic feed confuse and discourage the best students at first sight. Sometimes a cut is used which shows a picture of the cross section, and often a cut is given and no sectional drawing whatever is offered to aid the student. It is safe to assert that not one pupil in ten attempts to master by his own efforts such drawings as are usually placed before him. He gives up the attempt in disgust, if he makes one at all, and relies upon his instructor to show him "how it works." Text-books should contain in close proximity to each description not only a good cut but also a clear, simple, sectional drawing of the apparatus, with references in the text to the drawing as well as to the cut presented. These drawings should be so made that the significant facts of the apparatus will be firmly impressed upon the student's mind when he has completed the study of the text relating to the matter.

But teachers are at fault in this matter as well as text-books. A pupil is often directed to make a drawing of some piece of apparatus with which he has been working, the task being assigned without any preliminary assistance or instruction as to the essential parts to be drawn or the conventional methods of representing these parts. The pupil confronted by this problem usually does one of two things: either he openly or surreptitiously finds a drawing of some sort in his text-book or laboratory manual and then proceeds to copy it carefully into his notes, or else he produces by his own efforts a drawing out of proportion, with emphasis placed upon the wrong features, and with many minor but necessary details omitted. In either case it is safe to assume that he goes away and at once forgets what manner of thing it was that he drew. Later upon written test his instructor is surprised and pained to find that the student either fails entirely when called upon to illustrate his description of the apparatus in question or else his drawing and description do not agree. Examinations for state certificates prove that this deficiency is widespread and that inaccurate and incomplete drawings are the rule and not the exception. Every physics teacher will testify that pupils are prone to confuse the lift pump and the force pump, to omit valves, to place levers so that the motion produced will be incorrect, to form ropes and pulleys into impossible combinations, to produce wild tangles of wire in electrical apparatus, and to fail utterly to trace the path of a ray of light through a lens. In chemistry the drawings presented will not show the actual arrangement or will fail to show stopcocks, rubber connections, corks, etc., without which the apparatus would be useless. This condition points to vague ideas and a weak grasp upon essential principles, and demands vigorous treatment upon the part of our science teachers if improvement is to be secured.

It is almost axiomatic to affirm that no student can fully understand the apparatus he is studying unless he can make a simple but accurate drawing to illustrate its construction and operation. Graphic illustrations are helpful in any subject but nowhere are they of more value than in the sciences. Pupils should be taught to form correct visual images of the cross-section of the apparatus they are using. Without doubt one of the chief reasons why principles of physics which relate to common things such as the ordinary lift pump are so slowly apprehended is that plain drawings are not presented for study nor

is aid given the student so that he can make such drawings. Students may use a simple air pump or gas generator for months and be unable to explain its mode of operation or make an intelligible drawing of it. Hazy and erroneous ideas can often be located and corrected in no other way so well as by requiring the student to make a drawing of the apparatus. If the student secures by daily practice the ability to draw accurately and rapidly the apparatus he is using, he will receive much valuable training in designing new combinations to secure some desired end. Finally, the consciousness on the part of the pupil that he can draw some troublesome arrangement or complicated apparatus greatly increases his confidence in his ability to describe it lucidly and accurately when necessary.

The first essential of good laboratory drawings is rigorous simplicity. Only parts essential to a clear understanding of the principle under consideration should be shown. In accordance with this idea, pupils should be taught to omit detailed drawings of levers and other devices to produce motion, the direction of motion being shown by arrows. Dynamos and batteries should be represented by conventional signs or omitted altogether. Any detail of structure shown in the commercial model which is unnecessary and which may serve to distract the pupil's attention should be omitted. The constant aim should be to strip everything of that which is incidental or unnecessary and concentrate the attention upon the principle in hand. Students should not be allowed to make excellent drawings of ring stands or burette clamps and then fail to show clearly the apparatus of which these are merely the supports.

A second essential is that drawings be cross-sections of the apparatus used. Pupils should be directed to imagine that a vertical plane is passed through the middle of the apparatus and then the sheet of paper is put in the place of this plane. Lines are then traced where the paper would touch the intersected apparatus. The teacher must explain this principle carefully and enforce it by much practice. Special directions will be needed in both physics and chemistry. For example, students must be shown that the bottoms of bottles, flasks, cans, etc., must be represented by straight lines; that rubber stoppers, corks, etc., may be indicated by cross-hatching; that there should be no lines for the tops of open bottles, flasks, jars, etc., inasmuch as these would not show in a cross-section; that the plane should cut all essential parts; and that no shading should be used.

Laboratory drawings should also be well proportioned. The relative size of the various parts of the apparatus should be clearly shown. The drawings should indicate how the various parts can best be arranged so as to require a minimum of glass tubing, rubber connections, wire, etc., to assemble them. This is primarily a matter for the teacher to illustrate in setting up his model apparatus before the class, but attention to it in the drawings made will serve to impress it upon the pupil's mind.

If it is granted that the ability to make plain, accurate cross-section drawings is very desirable, the means to be used in securing this ability merits consideration. The teacher in science will look in vain to the regular high school work in elementary and advanced drawing for much real help along this line. To secure satisfactory results the teacher must give supplementary instruction and training. It would be well to spend one or more laboratory periods near the beginning of the year in explaining how cross-section drawings should be made and in training the class to make good drawings by actual work with some given arrangement of apparatus. The time thus spent will be more than regained by the increased efficiency shown in this respect during the balance of the year. Every new principle to be illustrated by apparatus in the laboratory should be developed in the class room first by the use of a good sectional diagram on the board explained before the class by the instructor. Results will be best if the instructor skillfully builds up this drawing as he explains the construction. The drawing should evidently be made in the order in which the parts would be assembled or the instrument studied in the laboratory. In addition to board work by the instructor, much free hand sketching on the board and upon paper should be done by the class.

It may be suggested in closing that whenever we find that our students have inaccurate notions in regard to certain apparatus and its purpose, it will be well for us to determine whether we are teaching them how to draw what they study. In many cases some systematic work along the lines indicated above will be found very beneficial in clarifying and strengthening a pupil's knowledge.

THE SIGNIFICANCE OF THE REQUIREMENTS IN PHYSICS OF THE COLLEGE ENTRANCE EXAMINATION BOARD.[1]

By John F. Woodhull, Ph.D.,
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Physics is poorly taught in the colleges—quite as poorly taught in the colleges as in the high schools.

The duty of fitting the colleges for the high school graduates is quite as urgent as that of fitting the high school pupils for the colleges.

The colleges should prepare teachers of physics for the high schools but those who are most recently come from college are the most conspicuously unfit for teaching high school pupils.

These conditions must be kept in mind when preparing a syllabus of work for high schools.

By years of contact with high school problems some college graduates who are capable of adaptation have been transmuted into good teachers of physics. Such were the six high school men chosen by the College Entrance Examination Board to prepare the present syllabus.

With the clear understanding of conditions as they exist at present, they did the wisest thing that could be done under the circumstances. Education must be a conservative thing. Changes must not amount to a revolution. What these men did was a step in the right direction, and a sufficiently long step to be practicable, yet it is not the end of progress toward better things, it is merely a small beginning.

It is worth while to remind ourselves that the so-called College Entrance Board is not in reality a college entrance board. It is merely an examination bureau. It has gradually succeeded in getting its certificate accepted for entrance by most of the colleges just as the regents of the state of New York have done, but unlike the regents this board could not continue to exist if its requirements on examinations did not conform pretty closely to what the colleges demand. Its real name is, The College Entrance Examination Board—with the emphasis on examination. By calling it as we do, for short, the College Entrance Board, we sometimes mislead ourselves as to its importance. It cannot be expected to be so much a leader as a follower in educa-

[1]Read before the New York Schoolmasters Association Nov. 13, 1909.

tional progress. The committee was well aware of the restricted functions of the board and therefore committed it to no embarrassing position.

On the other hand this board as an exponent of the colleges makes it known that the colleges have now for the first time become ready to recognize the fact that the high schools are better able than the colleges to determine what should be expected of high school pupils. It behooves the high schools to get ready for some constructive work and to take a step forward each year in the matter of these physics requirements.

Let us first examine the report of this committee and see what changes they have made and then consider the next changes which should be contemplated.

The greatest hindrance to good teaching is not the syllabus. That like all codes of law is capable of many interpretations and may by exegesis be made to justify all practices; not the examiners, for under the regime of the College Entrance Examination Board they have usually been broad-minded men, and at any rate their sole function is to frame questions—which again are capable of many interpretations—but the real block upon high school teaching are the readers and raters of examination papers and laboratory notebooks. These men are a law unto themselves and their rules of conduct have never been published. It is certain that they have not the same outlook upon education as the board has or as its examiners have, and it is equally certain that they exert ten times as much power as either the board of the examiners in shaping the high school work.

The most unsatisfactory part of the physics requirement (or supposed requirement) has been the laboratory work. I say, supposed requirement, for probably no one in any position to *require* has ever wished for such extremities of mathematical frivolities as many of the recent offshoots from the colleges have vainly tried to implant in high school laboratories. As usual the disciples in trying to imitate their masters have greatly exaggerated their foibles. Unconscious that what is bad for college students is worse for high school pupils, they have seized upon what was either evil or without significance in their own college laboratory work and wrung interminable changes upon it for high school pupils.

The real laboratory requirements, however, as executed by the readers have been a great handicap to good teaching. The reader's interpretation of what an orthodox laboratory notebook

should be has never been authorized and never published. From the record of successes and failures of their candidates the high school teachers have inferred that the readers insisted upon the notebooks containing much of that which I should regard as not physics at all. The high school teachers have likewise found that small credit would be given to that which this committee declares is the very aim of laboratory work, namely, "to supplement the pupil's fund of concrete knowledge and to furnish forceful illustrations of fundamental principles and their practical applications. To perform exercises such as yield results capable of ready interpretation and free from the disguise of unintelligible units . . . Unnecessary mathematical difficulties being avoided and care being exercised to prevent the student's losing sight of the concrete facts, in the multiplication of symbols."

Most of the unjust rating of candidates for admission to college has been the result of the absurd attempt of human minds not widely trained to estimate the value of the laboratory work of a total stranger by glancing over in four minutes the notes which required a year in the writing. No demand has been more insistent than that this abuse be stopped. A majority of that committee of high school men went into the meeting fully determined that it must be stopped.

The first and most important reform which they have brought about is that The College Entrance Examination Board will no longer undertake the marking or examination of laboratory notebooks. The high school teacher is to certificate the laboratory work wholly according to his best judgment. It is no longer indicated to him by the syllabus what shall be the nature of the laboratory work.

It will doubtless need to be that which will best help the pupils to pass the topical examinations, but I estimate that fully one half of the time now given to laboratory work in many of the best schools does nothing toward helping the students to pass the examination questions and does nothing toward giving them a clearer understanding of physical principles. Since half of the time given to the study of physics in these schools is devoted to the laboratory the way is now open for them to save one quarter of their efforts for work that is worth while. If the committee had done nothing else than to abolish the rating of laboratory notebooks by readers, we could easily have been sat-

isfied for one year. For now there is no reason why a capable teacher may not make all of his laboratory teaching fruitful. Before, much of it had to be barren by specific requirement.

In the number of topics which appear in the syllabus the committee left a larger prescription than the high school teacher can do well—but it probably did all the pruning that it was safe to do this time. The most efficient teachers are the ones who will see the largest possibilities in each topic for instruction and will therefore feel the greatest burden. Many complaints from such persons are coming in, and now that the high school teachers have their hand in they will doubtless bring about a reduction in the number of topics when the syllabus is next revised. The committee probably had in mind that the field was too broad to cover thoroughly when they made their second recommendation which reads as follows: "We urge upon those who prepare the examination questions that these be so planned that students who have received fair preparation on the work as here outlined may reasonably be expected to pass." This sounds very different from the boast which has been made that after casting out one-third of each class during the four years of the high school course we serve the cause still farther by knocking out half of those who try for admission to college. This committee of high school men speaks with some of the missionary spirit appropriate to those who in the service of the people are trying to extend the benefits of education.

It is not expected, I presume, that I should discuss with this audience the specific additions or omissions in the list of topics. You care not, I presume, whether $f=ma$ is included or excluded. Yet I must tell you that a considerable amount of such pestiferous knowledge has been removed from the syllabus.

Now a few words as to the future. I am speaking to an audience composed of principals of secondary schools and of the teachers of all the various high school subjects. We all have, however, a common cause and it is upon our common problems that I wish to speak. The same thing is the matter with physics teaching as with all other subjects. I saw a young college graduate teaching a high school class in English. She had detained them six weeks on the first third of Ivanhoe, using the method of the higher criticism. She informed me that it was the only way to get them ready for college. Which is true. It is also true that it was the only way she knew for teaching Ivanhoe and for

both of these the colleges are to blame, but, worse than all, when after attending educational meetings and reading educational literature and having a few years' more contact with the high school pupils she learns a better way of teaching Ivanhoe, she will be prevented by college requirements from using it, unless the high school men reform the requirements.

I saw another college graduate teaching botany in a Western high school—a school accredited by the state university. They had for many weeks been examining plant cells with a compound microscope and filling many pages of notebooks with drawings of them. I asked if so much of this work were desirable. She said, "No, but the university inspector requires it." There was lying upon her desk a copy of Bailey's Botany, full of useful knowledge such as those high school pupils need. I asked her if it would not be better to give them that. She said, "The university would not approve it." It perhaps should be said in justice to the universities that such approval or disapproval generally comes from a minor officer—but so long as the universities allow their minor officers to act as their spokesmen they cannot escape the charge of being either ignorant of or indifferent to the cause of secondary education. These are not peculiar cases. There is no subject which can be well taught in the high schools to-day for the simple reason that the colleges prevent it.

The case of high school physics to-day is analogous to a situation which obtained for Latin thirty years ago. It was the practice of some teachers then to spend two or three weeks previous to the teaching of Virgil in training pupils in the rules of the grammar which applied to scanning poetry. These rules with their exceptions numbered several scores. They were learned, "and stored in memory so that pupils might have them to use when they came to the business of reading Virgil." It often happened that Virgil was taught by another teacher and frequently this teacher made no mention of scanning but of course if he did he discovered no ability in the class to use those rules of grammar. This is precisely the case of high school physics to-day. The high school teacher feels obliged to teach what might be very well called the rules of physical grammar. Some teachers know that language teaching must come before grammar, but the real powers that prescribe the work for all teachers, good and bad alike have not yet got beyond the a-b, ab method of teaching. After compelling the high school teachers

to teach by an impossible method and one long since discredited in other subjects, these college people are surprised to find that the high school pupils do not understand physics.

This topical syllabus is a list of principles. A few teachers are urging that some practical applications be added, as if to say, "Some few sentences from good literature should be appended to interpret the rules of grammar," but for every one teacher who dares to suggest this a dozen will respond, we have not time to teach applications, the list of principles is so long and in any case we cannot teach applications before we teach principles. And the readers of examination papers on their part say, "We cannot rate questions on applications of principles; they are too indefinite; we must mark on the principles and particularly on the mathematical work, for that is definite." The examiners have on several occasions attempted to encourage teachers to enrich their work in physics by asking some general question about, say, the heating of buildings. Such questions cause great perplexity to the readers and they decide by conference what specific answer will be given full credit and allow nothing for any other. As, for example, if the candidate explains specific heat in answer to the above question it will be rated correct but a paragraph or a page of other replies, however intelligent, will be rejected without credit.

The makers of the syllabus and of the examination questions are still further moved to eliminate applications because applications in physics must of necessity vary with locality, occupation, sex of pupils, etc., and the whole business of standardizing physics and bringing about a glorious uniformity runs counter to this.

The truth is that the so-called fundamental doctrines of physics are fundamental neither to the science of physics nor to the teaching of that science. Our doctrines in physics are no more fundamental than they are in religion. We change them oftener. Neither are these principles any more help to the *teaching* of physics than Calvinism is in the *teaching* of religion.

I hope the high school men, pushed on by the public which is behind them, will gradually change that list of principles in physics to a list of physical phenomena or rather experiences which one may encounter in life and of which the ordinary man desires an explanation. And I predict that when high school pupils have been taught to notice the experiences of

a physical sort which their lives present to them and to classify them so that one fact throws light on another, they will come to the college instruction with a gratifying ability to grasp physical principles. But it must be their own life experiences, found outside of the conventional laboratory—and not such as may be prescribed for them by some central body, whose only anxiety is to rate and uniformitize the contents of pupils' minds.

Our besetting sin in physics teaching is something which goes under the euphonious name of thoroughness. We have a block-measuring mania in some parts of the country.

Pupils are required to measure the four edges representing the length of a block of wood to the tenth of a millimeter which is 1/254 part of an inch. These are to be averaged and the result taken as the true length. In like manner, the four edges representing the width and the four representing the thickness are to be treated and then these results are to be multiplied together and the volume recorded in terms which represent less than 1/1,000,000 of a cu. in. The pupils know that the exercise lacks common sense, since the measuring sticks are crude, the edge of the blocks are battered, their faces are not perfect planes, and the human eye is incapable of reading such microscopic dimensions. If such training has any effect it would seem rather to disqualify them for making useful measurements like the length of the school room or estimating its height. Consider the years spent in drill. The elementary school-teacher devotes a very large portion of time to drill upon mathematics. The high school teacher thinks the grammar school training is defective and he takes up the drill upon mathematics. The college teacher believes the high school training is defective and he attempts the same drill. In this series of onslaughts upon the students the college teacher effects the least results and the elementary school-teachers' work is the most efficient. Investigations show that drill in mathematics beyond a certain few years of it have no effect upon the pupils' abilities.

The percentage of high school pupils studying mathematics increased during the fifteen years, 1890-1905, from 66% to 88% and during the same time the percentage of those studying physics decreased from 22% to 15% and as though that were not bad enough we have tried to convert physics into mathematics, and we have been told that this was in the interests of thoroughness.

I predict that school pupils will some time have the privilege which we, who have got beyond the domination of our schoolmasters, enjoy of learning many things with the intention of forgetting them immediately—more than half of the subject of physics should be so treated. If we adults had to learn thoroughly and retain for somebody's examination one tenth of that which we find it both delightful and profitable to learn each day we should be miserable.

These children know wherein we wrong them and the best of them are longing for the time when they may rise above us. Is there any reason why we should not be willing that the children should learn by the same method which we find profitable? We learn by the smattering method—first a passing introduction—a mere fleeting impression—then time for the ideas to incubate, and after a while another meeting with these ideas from perhaps a different standpoint followed by casual meetings at intervals. The method of the daily newspaper reader is producing clear thinkers.

Another privilege which every person young and old rightly prizes is that of learning something about many things which we cannot understand and ought not to be examined upon. The requirements in physics should be cut down so that there may be time to thus enrich the work in physics by lectures and outside reading. We have preached against mere memoriter work until our preaching has taken some effect in high school teaching but we continue to give examinations which no amount of general intelligence would enable one to pass. No faculty nor body of graduate students could pass the entrance examination—only those can pass them who have recently crammed their memories with the conventional material which appears nowhere else than in examinations. We have heard that high schools are deteriorating because their pupils do not pass entrance examinations to college, West Point Military Academy, and the like. I undertake to say that the more rational our instruction becomes the less will our pupils be able to pass the examinations as now given. Perhaps it is a commendation of the high schools that the secretary of the College Entrance Examination Board notes in his last report that a larger number of candidates than usual failed to pass the examinations.

An eminent authority says, "Really educated people are just those who have forgotten more than others." It is essential to

good thinking that one should not retain in memory much unorganized and useless material. Ideas sometimes require a long period of incubation. Inner intellectual life of extraordinary wealth is often hidden because of lack of means of adequate verbal expression or perhaps lack of the special gift of passing written examinations.

It might be well for this association to appoint a commission to investigate the returns now on file with the secretary of the College Entrance Examination Board. It is idle for individual pupils or teachers to attempt the matter of appeal but it is the duty of the secondary schools to have a commission for this purpose. I believe there are just as good persons kept out of college as are admitted by these examinations. The high school-teachers know before the examinations who are qualified to enter college and they know what ones have injustice done them by the examinations. It is the duty of someone to defend the pupils and to defend the good name of the schools.

The calorific value of ten tons of peat, freshly dug and untreated, is assumed to be equal that of one ton of ordinary coal. There are in Ireland 2,800,000 acres of unutilized bog land with a depth varying from 16 to 30 feet. There are 18,231 tons of peat to one acre of bog and this would be equivalent to 1,823 tons of coal. Accordingly, the total calorific value of the bog lands of Ireland is estimated as equal to that of 5,104,000,000 tons of coal. Moreover, at least half of this quantity could be conservatively counted on as available ultimately for steam-raising purposes. Peat fuel equal to coal could be produced in selected localities in Ireland at $1.75 per ton. Some authorities maintain that the peat bogs are reproductive, and that in some cases the bogs increase in thickness 5 or 6 inches a year, according to the nature of the flora. This is simply theory, however, and there is doubt as to whether they are recuperative at all. In case reproduction be at all feasible, the life of the bogs as a fuel reserve would, under adequate conservation, be capable of indefinite extension.

EXPLANATION OF THE TERM FOURTH DIMENSION.

By G. A. Miller,
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The term, fourth dimension, should be used in such a way as to be in accord with the common usage of the terms one, two, and three dimensions. Hence we shall first inquire how the latter terms are generally used. All agree that the straight line has only one dimension if the point is regarded as the element. The number of points on a straight line is evidently equal to the number of line segments which lie on this line and have a common end point. Hence a straight line has also only one dimension when these line segments are the elements. On the contrary, a straight line has two dimensions if the line segments on it are regarded as the elements. The truth of this statement will appear more clearly in what follows. It appears, however, very evident that there are more line segments than points on a straight line, since we can establish a one to one correspondence between all these points and the segments which have a common end-point. In this one to one correspondence the segment of zero length may be said to correspond to the common end-point.

The points on a line are commonly placed in a one to one correspondence with all the real numbers by means of an arrangement in which the numbers represent the distances of the points from a fixed point. Such an arrangement is suggested in the following diagram:

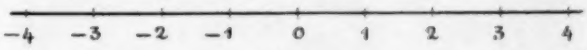

Fig. 1

In view of this one to one correspondence between these points and the real numbers, it is customary to say that the real numbers have only one dimension. As the establishment of a one to one correspondence between points and numbers has been so extremely fruitful in the development of mathematics, it was natural that common concepts relating to these fundamental notions should receive common names.

The points in the plane are commonly placed in a one to one correspondence with all possible pairs of real numbers by means of an arrangement in which the numbers represent the distances of the points from two fixed lines cutting each other at right

angles. Such an arrangement is suggested by the following diagram:

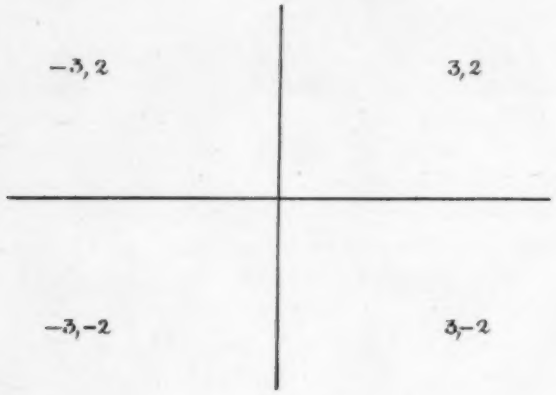

Fig. 2

Hence we say the totality of the pairs of real numbers has two dimensions. Every possible pair of real numbers may be uniquely represented by a line segment on a straight line if the first number of the pair represent the initial point of the segment and the second number represent the length of the segment, the sign of this number indicating the direction. For instance, the pairs (—3, 2), (3, —2) may be represented as follows:

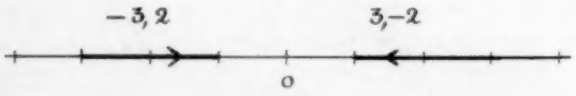

Fig. 3

Since the possible pairs of real numbers can be placed in a one to one correspondence both with the points in the plane and also with the line segments on a given line, it is natural to say that the number of line segments on a straight line is equivalent to the number of points in the plane. Since the plane is said to have two dimensions when the point is taken as the element, the line should be said to have two dimensions if the line segments are regarded as the elements. The fact which is to be emphasized in this connection is that the dimension of a geometric figure is not an intrinsic property of the figure but it depends upon what is regarded as the element.

By extending the notions mentioned above we may associate with each triplet of real numbers a point in space, and with each

point in space a triplet of real numbers in such a way that there is a one to one correspondence between the triplets and the points of space. Hence the possible triplets are regarded as having three dimensions. There is a one to one correspondence between the possible circles in a plane and the triplets of real numbers if the first two numbers of the triplets indicate the centers of the circles while the last represents the length of the circumference, the sign indicating the direction in which the circumference is described. In view of this one to one correspondence between the points of space, the triplets of real numbers, and the circles in a plane, it is natural to regard the plane as having three dimensions if the elements are circles.

Just as the totality of sets of three real numbers is said to have three dimensions so the totality of sets of four real numbers is said to have four dimensions. Since a set of two real numbers represents a point in a plane, or a point in a two dimensional space, and a set of three real numbers represents a point in a three dimensional space, a set of four real numbers may be said to represent a point in a four dimensional space. It is easy to establish a one to one correspondence between the sets of four real numbers and some geometrical figures. When this is done the smallest space in which these figures lie is said to be four dimensional when the figures are regarded as the elements.

For instance, all the possible circles which are parallel to a given plane constitute a four dimensional manifoldness, since the centers of these circles may be uniquely represented by the first three, in the possible sets of four real numbers, while the last numbers in the sets may represent the length of the circumferences, the sign indicating the direction in which the circumferences are described. We thus obtain a one to one correspondence between the possible sets of four real numbers and the totality of parallel circles. We shall show that such a correspondence may also be established between these sets and the totality of the squares in a plane. Hence we shall say that the plane has four dimensions if all the possible squares lying in it are considered as the elements, while it has two or three dimensions, respectively, as the points or the circles are taken as the elements.

To exhibit clearly that there is a one to one correspondence between the totality of sets of four real numbers and the totality of squares lying in a plane, it seems desirable to emphasize the fact that by definition an infinite multitude is one in which a part

is equivalent to the whole. That is, we can establish a one to one correspondence between the units of an infinite multitude and the units of a part of this multitude. Although this statement may appear strange at first it is nothing more than saying that the equation

$$y+1=\frac{1}{x}$$

can be solved uniquely for every positive value of y and that two different values of y always give rise to two different values of x. In other words, this equation establishes a one to one correspondence between the totality of positive numbers and the numbers representing the corresponding values of x. It is evident that the limits of x, when y represents successively all the positive numbers, are 0 and 1. Hence we have to say that there is a one to one correspondence between all the positive numbers (including those from 0 to 1) and the numbers extending from 0 to 1. By means of the equation

$$y+1=\frac{a}{x}$$

we can similarly establish a one to one correspondence between the numbers from 0 to a and all the positive numbers, if a is any positive number whatever.

With this example before us it is easy to see that there is a one to one correspondence between the possible squares in the plane and the totality of sets of four real numbers. The first two numbers of such a set may represent the center of the square and the third may represent its perimeter, the sign indicating the direction in which the perimeter is described. The fourth numbers in all the possible sets may represent uniquely some angle from 0 to 90 made by a side of the square with a fixed line. To do this it is only necessary to establish a one to one correspondence between all the possible real numbers and the numbers from 0 to 90. This may be done in many ways; for instance, we may establish a one to one correspondence between all the positive numbers (y) and the numbers (x) from 0 to 45 by means of the equation

$$x=\frac{45}{y+1}$$

The negative numbers may then be placed in a one to one correspondence with all the numbers from 45 to 90 by the rule that the number corresponding to a negative number is 45 plus the one corresponding to a positive number (y) having the same absolute value. That is, we determine x by means of the first

or second of the following equations as y is positive or negative:

$$x=\frac{45}{y+1} \qquad x=\frac{45}{1-y}+45$$

The method of argument followed to exhibit that the plane has four dimensions if the squares are regarded as its elements may also be employed to show that space has four dimensions if the spheres are regarded as its elements. In fact, the first three numbers of every set of four real numbers may be regarded as the centers of spheres and the fourth numbers may be placed in a one to one correspondence with the positive numbers, and then these positive numbers may be used to represent the length of the radii of the possible spheres.

From the fact that there is a one to one correspondence between the points of a line and those of any finite line segment, it results that any line segment, besides the zero segment, is one dimensional if its points are regarded as its element and that it is two dimensional if its line segments are regarded as the elements. Similarly any arbitrary portion of the plane is four dimensional if the squares in it are regarded as its elements and an arbitrary portion of space is four dimensional if the totality of the spheres lying in it is regarded as constituting the elements. The fundamental idea of n dimensions is a one to one correspondence between its elements and the totality of sets of n real numbers. From this standpoint the fourth dimension has very much in common with the smaller number of dimensions and these common properties seem to offer the most fertile field of study. This conclusion is in accord with Poincaré's observations that "the great advances of the past were made by the union of two sciences, by observing similarity of form regardless of dissimilarity of matter, by modeling the one after the other."*

In the process of utilizing the advances of one science in other related sciences it frequently has been helpful to assign to a term broader meaning. In fact, in the article to which reference has been made, Poincaré said that in a certain sense mathematics is the art of giving the same name to different things. The use of the term dimension for sets of numbers is in harmony with this fertile tendency which helps to clarify and systematize our mathematical knowledge. The term fourth dimension is used because it emphasizes the relation between the more complex and the simpler concepts. By observing these relations progress becomes easier and extensive developments present fewer difficulties.

*Poincaré Bulletin des Sciences Mathématiques, vol. 32 (1908), p. 178.

MEASURING INSTRUMENTS OF LONG AGO.

WILLIAM E. STARK,

Ethical Culture School, New York.

A few years since, in searching for practical problems to vitalize high school work in algebra and geometry, I looked through many artisan's manuals. In the course of this search, I found in a carpenter's handbook of fifty or sixty years ago descriptions of several obsolete instruments, which furnished excellent illustrations of simple geometric principles. Having struck "pay ore," after much profitless digging, I determined to follow the vein. The task has been altogether delightful and has yielded not a little material which will add life to the stereotyped five books of geometry. The facts and illustrations given below are taken from the collection of books and instruments belonging to Professor David Eugene Smith of Columbia University. Most of the books referred to are of the sixteenth and seventeenth centuries.

The chief advantage of these old instruments for educational purposes lies in their simplicity. Modern science demands such a high degree of precision that most measuring instruments of to-day are too complex to be easily understood by the young student. The devices for increasing the accuracy or extending the field of use of one of these modern instruments obscure its fundamental principle. The engineer's transit is many instruments in one.

The primitive instruments were intended for a single purpose and were based on a single principle. To be sure, there are in the books referred to many examples of the tendency to combine instruments and many an author undertakes to show how his invention may be used to do the work of all others. Nevertheless, the simplest devices will be readily grasped by the youngest high school pupils and all that I refer to will be understood by the older students.

The chief value of this material is, I think, for giving concrete illustrations to the abstract theorems. Even the impractical devices will be of service in stimulating the student to follow the inventor's thought and to discover the sources of error in his process. When an author proposes to determine the distance to a point more than a thousand feet away, using a base line

five feet long, our student of geometry is likely to take some satisfaction in proving the inaccuracy of the method.

Some of the instruments may well be used for field problems and simple surveying. A good transit is beyond the reach of most schools but many of the instruments described below can be constructed easily and cheaply by a boy who has had some training in woodworking. A few illustrations of instruments

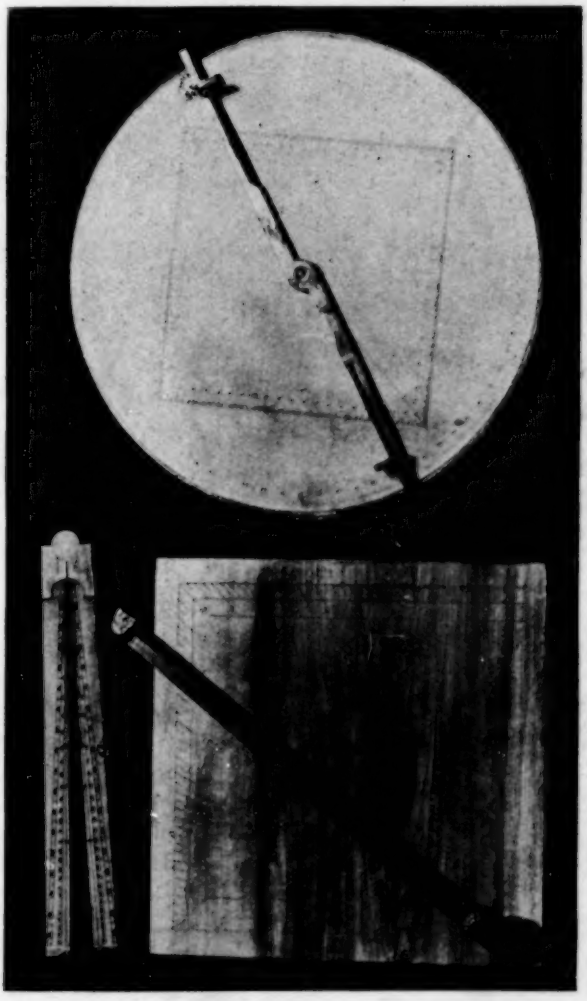

Figure I.

REPRODUCTIONS OF OLD MATHEMATICAL INSTRUMENTS, MADE BY PUPILS.

made by boys of the Ethical Culture School are shown. These instruments form a nucleus for a little mathematical museum, and are used occasionally for field practice. If it be suggested that the results obtained by such crude instruments will be too inaccurate to interest pupils in making the measurements, it should be remembered that surveyors and military engineers found these same devices accurate enough for all their work for some centuries. One who has used only precise instruments will be surprised at the degree of accuracy which it is possible to obtain after practice with such simple tools as those described.

Instruments for Measuring Distances.

The instruments used for making direct measurements of length in the sixteenth and seventeenth centuries were rods of various lengths, subdivided into smaller units, cords with knots

Figure II.

Surveyors' Tools. From *Opera Geometrica* by J. Buteo, Lyons, 1554.

Figure III.

USE OF RODS IN LINEAR MEASUREMENT. FROM *La Geometria Prattica* BY G. POMODORO, ROME, 1624.

to mark the divisions, and chains much like those still in use. The most noteworthy characteristic of these linear measures is the variability of the units. After reading a few pages of "equivalents" for use in turning units of land measure of one city into those in use in adjacent places, we wonder first why

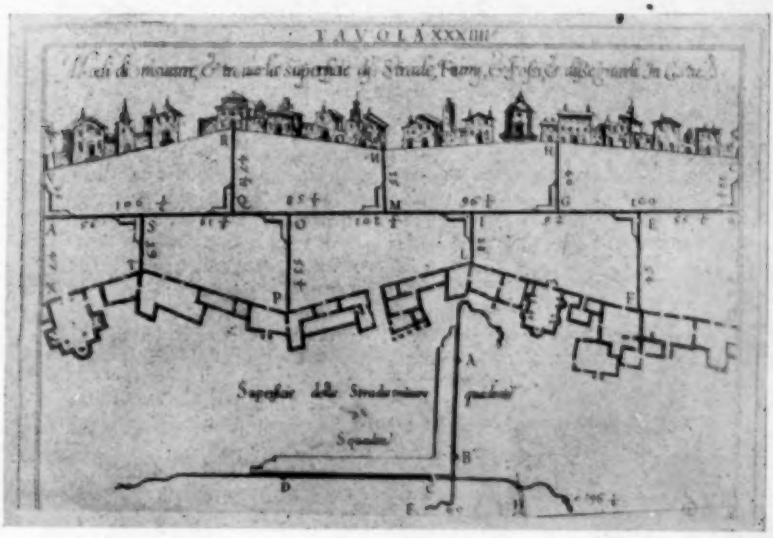

Figure IV.

USE OF SIMPLE SQUARE IN SURVEYING A STREET. FROM POMODORO. (SEE FIGURE III.)

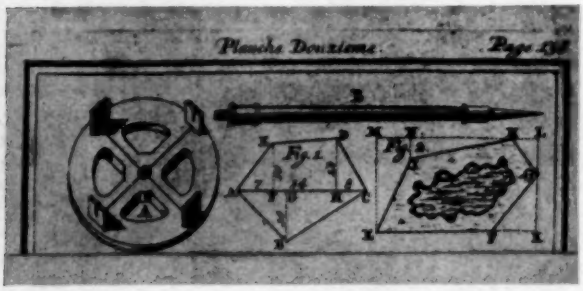

Figure V.

SURVEYOR'S CROSS. FROM BION, *Traité de la Construction et des Principaux Usages des Instruments de Mathematique.* THE HAGUE, 1723.

there should be complaint of present conditions and then why, when such notable progress has been made, it should be so difficult to take the last step.

Ottavio Fabri[1] in his treatise on the "squadra mobile" gives tables of linear and square measure for twenty-one cities, nearly all of them in Italy. He says: "In Ferrara land is measured by the *mozo* which is 66 poles (pertiche), 6 feet and 8 inches long and 20 poles wide. In the measures of Ferrera these 66 poles, 6 feet and 8 inches are equivalent to 116 poles, 4 feet, 8½ inches in the measures of Rovigo, and the 20 poles of the width are equivalent to 35 poles and 2½ inches. The product of these is 4091 *tavole*, 3 feet, 2 inches, which is equivalent to 4 *campi*, 3 quarters, 101 *tavole*, $3^1/_6$ feet in the measures of Rovigo. The 66 poles, 6 feet and 8 inches in Ferrarese measures make 127 poles and 2 inches in Paduan units, and the 20 poles of the width make 38 poles, 8 inches Paduan. The product is $4841^1/_6$ *tavole* which is equivalent to 5 *campi*, 3 quarters, $11^1/_6$ *tavole*."

INSTRUMENTS FOR MEASURING ANGLES.

For laying off right angles, a tool like an ordinary carpenter's square was often used. Figure IV shows such a square attached to the measuring cord for use in laying out perpendicular offsets. Another device for the same purpose was a sort of cross mounted on a staff (see Fig. V.)

For measuring angles a graduated circle or circular arc was employed. In some instruments, the angle was read by means of a rule or alidade, pivoted at the center of the arc, in others

[1] L'uso della Squadra Mobile. Ottavio Fabri. Venice, 1598.

a plumb line hanging across the scale indicated the angle, while in still others, the reading was taken at the point of a magnetic needle. An example of each of these types is given in the following illustrations.

In Figure VI, 4, a simple form of astrolabe, a full circle graduated in degrees carries an alidade with sights through which

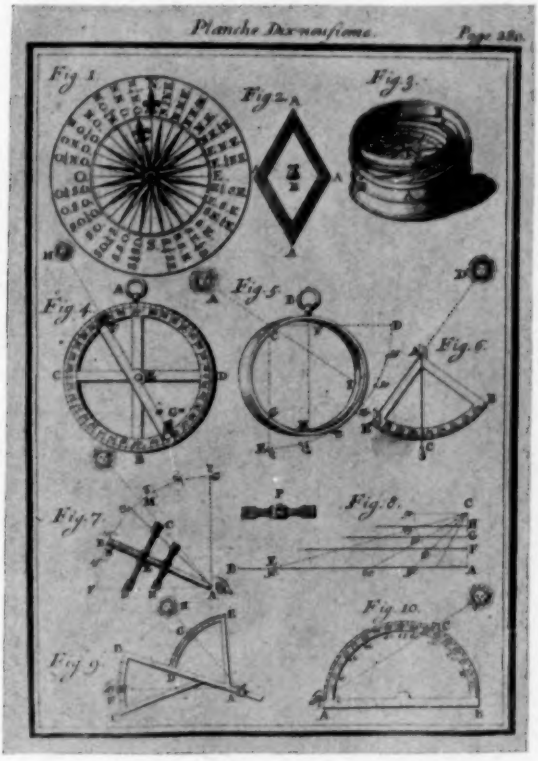

Figure VI.

NAVIGATION INSTRUMENTS. FROM BION. SEE NOTE UNDER FIGURE 5.

the observation is made. For measuring the altitude of the sun, the astrolabe was supported by the ring at the top, the plane of the instrument turned toward the sun and the alidade revolved until a beam of light passed through both sights. The edge of the alidade touching the scale then indicated the required angle. This instrument mounted on a staff could also be used in a horizontal plane.

In Figure VI, 6, is shown a quadrant with graduated arc and plumb line hung from the center. In the illustration an angular

altitude is being measured by aiming one of the bounding radii at the sun. The plumb line indicates the angle. The principle here exemplified is met by the student of geometry within a few days after the beginning of the course.

Figure VII is a cut of an instrument using a magnetic compass for measuring angles. The angle which any line makes with the magnetic meridian is obtained by setting the line of sight—which is parallel to the zero diameter of the compass dial—in coincidence with the required line, and noting the point on the dial indicated by one end of the needle. Of course any angle can be calculated when the bearings (angles with the meridian) of its sides are known. This instrument does not differ in principle from the surveyor's compass still in use.

In many instruments a semicircular arc was used for meas-

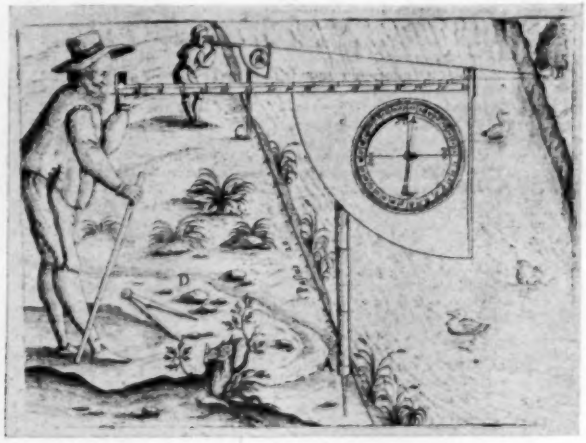

Figure VII.

EARLY FORM OF SURVEYOR'S COMPASS. FROM *De Quadrante Geometrico* BY CORNELIUS DE JUDAEIS, NUREMBERG, 1594.

uring angles (Figure VIII) and an author writing in 1612 dilates on the advantage of an octant (Figure IX). In the latter instrument the principle of the diagonal scale was used to subdivide the arc into ten-minute intervals.

A few inventors made use of the principle of measurement of inscribed angles. In the ring (Figure VI, 5), used in navigation for taking the altitude of the sun, a ray of light passed through a small hole, C, and fell on the inner surface of the ring opposite the hole. The scale on this inner surface was

formed by dividing the semi-circumference of the ring into ninety parts, the zero being at the same level as the hole which admitted the ray of light. Thus the tiny spot of light falling on the scale indicated the altitude.

An interesting device for increasing the accuracy of an in-

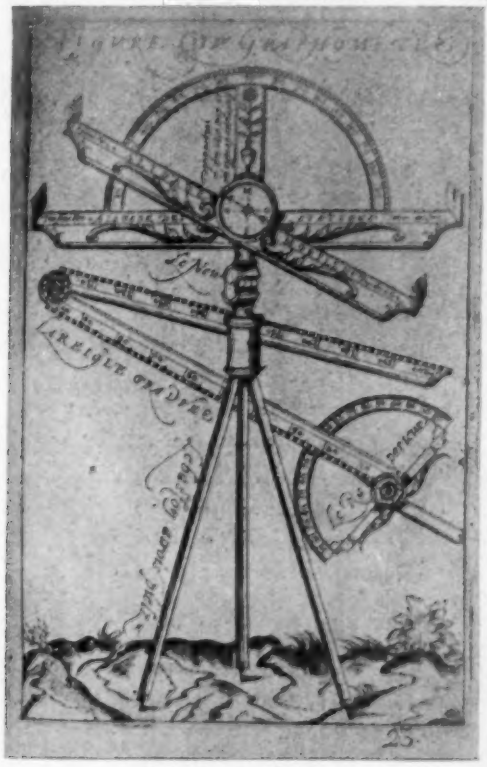

Figure VIII.

INSTRUMENT FOR MEASURING ANGLES. FROM *Declaration de l'Usage du Graphometre* BY PHILIPPE DANFRIE, PARIS, 1597.

strument without adding greatly to its weight is shown in the English quadrant (Figure VI, 9). This consists of a staff, AB, about two feet long, to which are attached two arcs, BC and DE as shown in the diagram. DE is divided into sixty divisions of one degree each. BC, which subtends an angle of thirty degrees, can, on account of its longer radius, be divided into spaces of five minutes each. Each arc carries a movable sight (F, G), and a third sight is placed at A, the center of both arcs. In practice, G is placed at any convenient division and F is moved until

the angle GAF coincides with the required angle, and the sum of the readings on the two arcs is taken. In measuring the altitude of the sun, F is held at the eye, A is aimed at the horizon and C is raised or lowered until a ray of light passing through G falls at A.

Mutio Oddi[2], writing at Milan in 1625, describes an instrument in which fixed sights are placed at convenient angles with each other, as shown in diagram in Figure X. The diameter OR bisects the quadrants CMA and SM✠, while IN makes an angle of thirty degrees with C✠. This device was to be used

Figure IX.

OCTANT FOR MEASURING ANGLES AND ESTIMATING DISTANCES BY PROPORTION. FROM *De Octantis* BY H. HOFMANN, JENA, 1612.

in laying out plots of land in the shape of regular polygons. The instrument was set either at the center or at a vertex of the proposed polygon, where by selecting the proper sights, the observer could lay out the required angle for many of the polygons. When an angle could not be obtained directly from the instrument, the nearest angle was laid out and corrected by means of a table of tangents given by the author. On reaching the theorem on the sum of the angles of a polygon, pupils will be interested in seeing how many of the regular polygons can be constructed by using angles taken from a figure like the one referred to.

[2]Dello Squadro, Trattato di Mutio Oddi da Urbino. Milan, 1625.

The *radius astronomicus*[3] used by astronomers for taking angular distances consisted of a staff with a cross-bar which could be moved along the staff, always remaining at right angles to it. Staff and cross-bar were graduated in small divisions and sights were placed on the cross-bar and at the end of the staff. For taking the angular distance between two stars, one end of the staff was held at the eye, and the cross-bar was moved until the sights, placed at equal distances on either side of the staff, were in line with the stars. Having read the number of divisions on the staff from the eye to the cross-bar and the number of divisions between the two sights on the cross-bar itself, the observer could look up the required angle in a table. The radius was provided with cross-bars of various lengths, the observer selecting the one best adapted to the size of the angle to be measured.

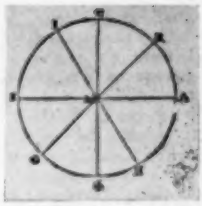

Figure X.

A later form of this instrument, used in navigation, had on each face of the staff a scale corresponding to one of the cross-bars, from which the angle could be read directly (Fig. VI, 7).

Instruments for Measuring Distances by Proportion.

The properties of similar figures are exemplified in many ingenious instruments and an indefinite number of problems. I shall mention first a few of the common problems solved without the aid of special instruments and then describe the devices invented for making similar measurements with greater accuracy.

Heights of towers and walls, widths of streams and moats and distances to inaccessible objects seem to have occupied much of the attention of practical mathematicians. Many of the authors refer to the ancient problems of calculating the height of a tower from the length of its shadow or from measurements to a mirror, placed on the ground, in which one sees the top of the tower (Fig. XI).

A quick but not very accurate method of measuring the width

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[3]Scripta Clarissimi Mathematici M. Johannis Regiomontani de Torqueto, Astrolabio Armillari, Regula magna Ptolemaica, Baculoque Astronomico, et Observationibus Cometarum Aucta necessariis *Johannis Schoneri* Carolostadii additionibus. Nuremberg, 1544.

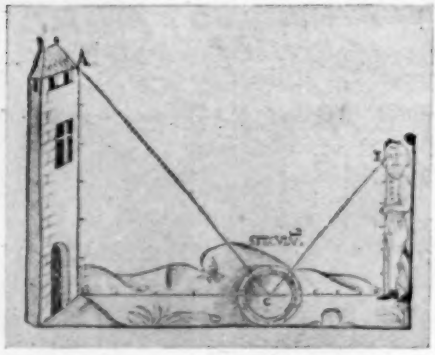


Figure XI.

USE OF MIRROR IN MEASURING A HEIGHT. FROM *De Re et Praxi Geometrica* BY ORONTIUS FINAEUS, PARIS, 1556.

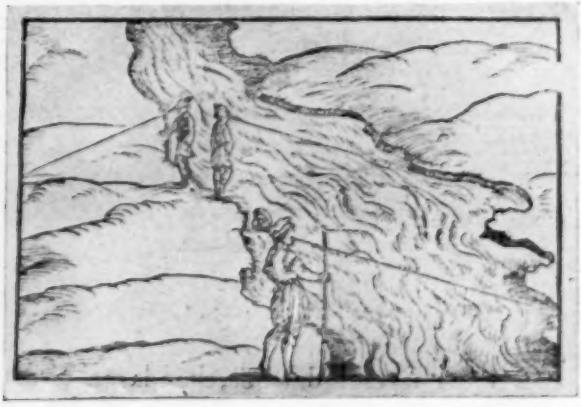


Figure XII.

MEASURING WIDTH OF STREAM. FROM *"Del Misurar con la Vista"* BY SILVIO BELLI, VENICE, 1569.

of a stream, was to plant a staff vertically on one bank, fasten near its top a shorter rod in such a position that it pointed at the opposite bank, then rotate the staff until the shorter rod pointed at a spot on the same bank as the staff. The required distance was then obtained by measuring from this spot to the foot of the staff. In Belli's geometry, 1569, this problem is illustrated by a cut in which the observer himself takes the place of the staff and the visor of his cap the sighting rod (Fig. XII). This problem will serve as an illustration of the equality of two triangles, having two angles and a side of one

respectively equal to the corresponding parts of the other. Pupils will be interested in finding out, by a drawing, how much error will result, if the staff is not quite perpendicular, or if the two points sighted at are not quite at the same level.

Much ingenuity is shown in devising figures in which the inaccessible distance is equal or bears an easily determined ratio to a line which can be measured directly. In the problem shown in Figure XIII, the distance of a ship off shore is measured by bisecting the angle formed at the observer's eye by his line of sight to point A on the vessel and to a point C on the shore

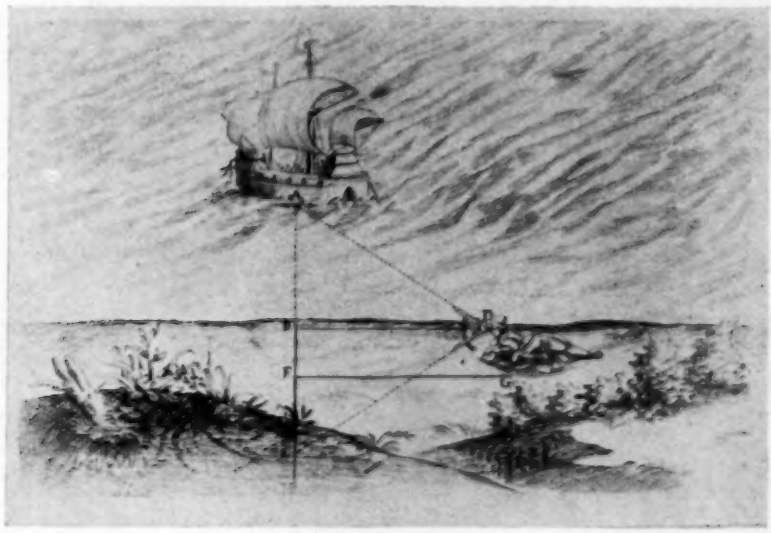


Figure XIII.

FROM *Apiaria Universae Philosophiae Mathematicae*, BY M. BETTINUS. BOLOGNA, 1645.

opposite A. The distance from C to the intersection of the bisector with AC is of course equal to the required distance.

In Figure XIV, EC is the required line. EB is laid off perpendicular to EC and at any convenient point, as B, the angle EBC is measured. CE is then extended making ED equal to EB and angle EDA is laid off equal to EBC. The distance from E to the point of intersection of DA with BE produced is equal to the distance sought.

Figure XV illustrates the measurement of the distance to a fortress from a point beyond the range of the cannon of the

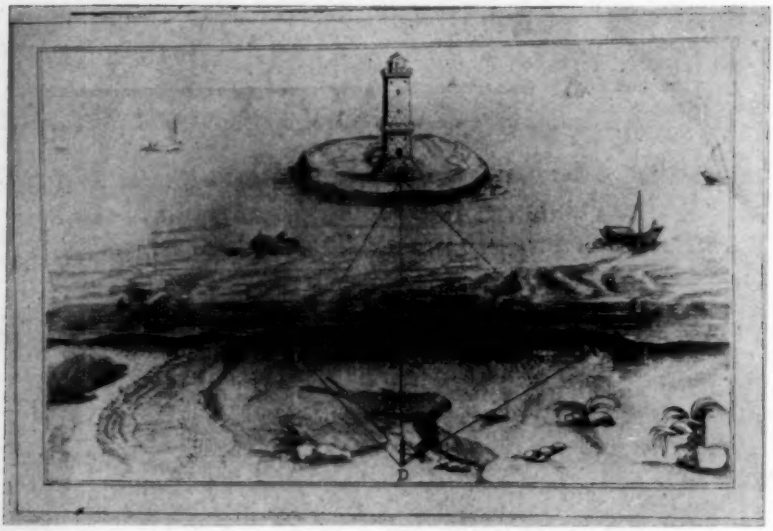


Figure XIV.
FROM BETTINUS. SEE NOTE UNDER FIGURE XIII.

garrison. EHI and HGI being right angles, the measurement of GH and GI is sufficient for the calculation of the result.

Figure XVI shows a method of finding the distance across a valley by measuring a base-line, CF, and calculating the ratio

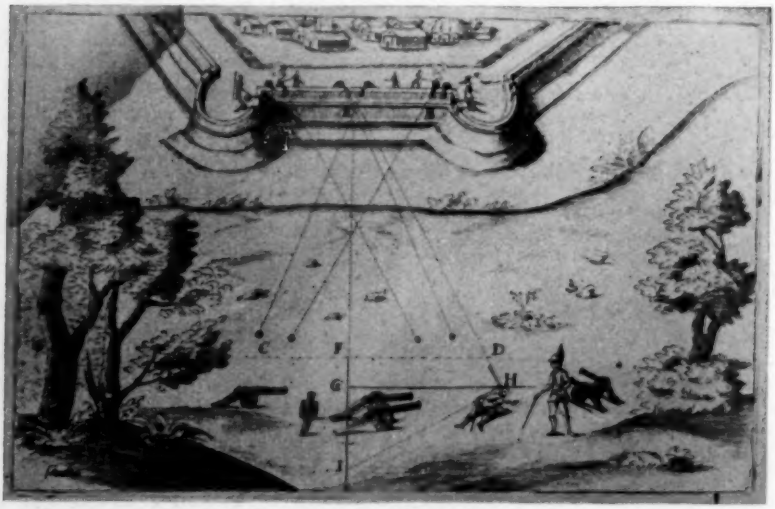


Figure XV.
FROM BETTINUS.

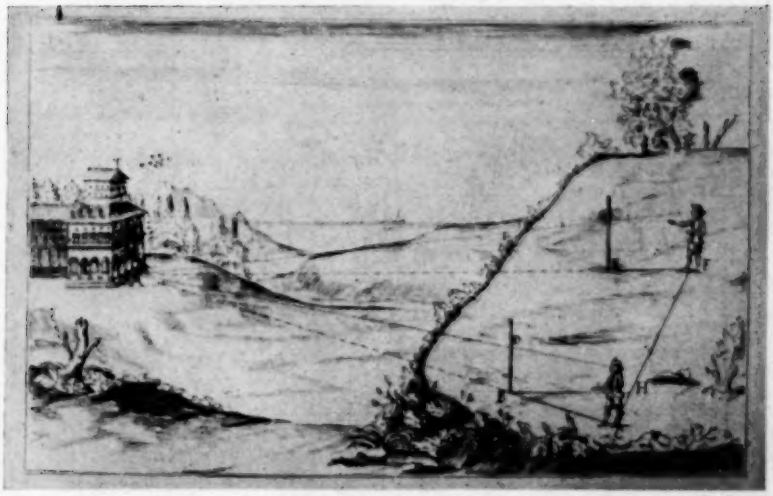


Figure XVI.

FROM BETTINUS.

of the sides of a small triangle similar to the large one of which the required distance is a side.

Some authors solve such problems by sighting at the inaccessible object from two ends of a measured base-line, representing the base-line and the directions of the sights by lines on paper, and scaling the required distance. Belli shows how this may be done when needed in military operations, by drawing on a drumhead (Fig. XVII).

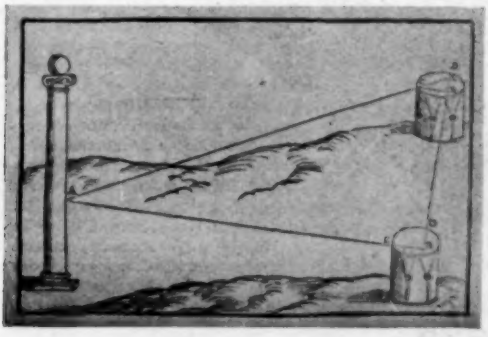


Figure XVII.

MEASURING A DISTANCE BY PROPORTION. FROM BELLI. SEE NOTE UNDER FIGURE XII.

The Geometric Square.

The instrument most commonly referred to in the sixteenth century authors and which appears to have been in use for at least five hundred years is the geometric square (quadratum geometricum). In its simplest form, it consisted of a square frame, two sides of which were marked with scales of equal parts, the number of divisions being usually twelve or some multiple of twelve. These scales, referring apparently to the measurement of altitudes by shadows, were called "umbra recta" and "umbra versa." At the corner of the square opposite the intersection of the scales was pivoted a sighting arm or alidade (Fig. XVIII). In the simplest problem, that of measuring the height of a tower, one edge of the square was held vertically and the alidade was aimed at the top of the tower. The reading where the edge of the alidade cut across the scale was then taken. If the reading was on the "umbra recta," the height of the tower was less than its distance from the observer; if on the "umbra versa," it was greater.

In the first case, assuming the reading to be five and the number of divisions on the scale twelve, the height of the tower

Figure XVIII.

Geometric Square. From Finaeus. See Note Under Figure XI.

was five twelfths of its distance; in the other, with the same reading, it was twelve fifths of the distance.

A quadrant was sometimes placed within the square for measuring angular altitudes, and a plumb line was often attached to one edge. Some forms had supporting staffs to which they could be fastened in either a horizontal or a vertical position.

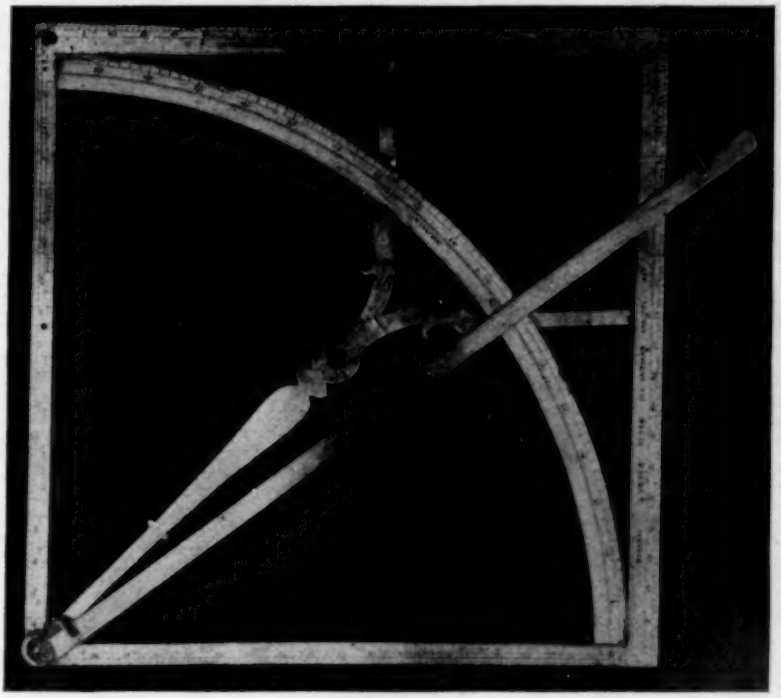


Figure XIX.

GEOMETRIC SQUARE OF THE SEVENTEENTH CENTURY, BELONGING TO PROFESSOR DAVID EUGENE SMITH OF TEACHERS' COLLEGE, NEW YORK.

The early instruments were made of wood and were three or four feet square, but the later ones were of metal. The one belonging to Professor Smith (Fig. XIX), dating from about 1600, is made of brass and is about twelve inches square. "Umbra versa" and "umbra recta" scales were often engraved on astrolabes. Chaucer, in his treatise on the Astrolabe, 1390, describes this feature, and explains how to solve several problems by means of it.

In another common form of the instrument, for use in measuring altitudes, the square was inscribed in a quadrant (Fig.

XXII). Instead of an alidade, the quadrant had sights attached to one of the radial sides and the reading was taken under a plumb line which hung from the center.

An interesting modification of the quadrant form of the geometric square (Fig. XXIII) was devised for the purpose of reducing a distance measured on a slope to a horizontal measurement. Near the center of the quadrant, BAC, which is made of wood or metal, a graduated straight edge is pivoted. The line EM, which represents the position of the straight edge when AB is vertical, is graduated in the same way as the straight edge, and a series of semicircles is drawn through the points of division as shown in the diagram. In use (Fig. XXIV) the side AC is held parallel to the slope and the straight edge is allowed take its natural position. Suppose the distance on the slope to be 70 feet. Then the reading of the straight edge where it cuts the semicircle which passes through 70 on scale EM gives the horizontal distance. If the slope distance is too long to be applied directly to the scale EM, it is divided by two or three for example, and the reading is multiplied by the same number.

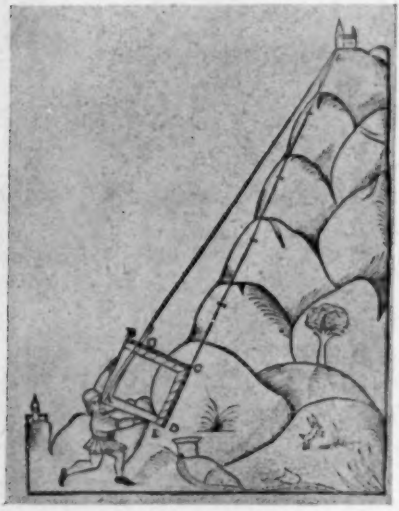


Figure XX.
USE OF GEOMETRIC SQUARE. FROM FINAEUS.

A general lack of skill in computation is implied in the proud claims of many authors that with their instruments all calculation is avoided. Belli's square (Figure XXV) was solid and ruled like a checker board. Holes were bored at the graduation marks along two sides of the square, into which the pivot of the alidade could be thrust. With this device it was possible to make one side of the triangle formed on the instrument correspond in length to the base line, and so to read the required distance directly.

The most interesting problem solved by means of the square

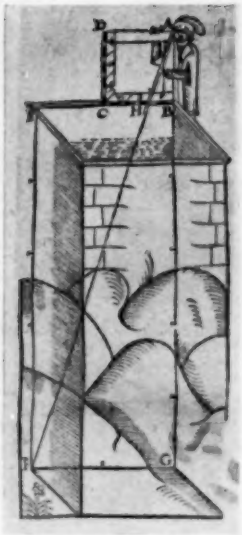


Figure XXI.

USE OF GEOMETRIC SQUARE IN MEASURING DEPTH OF WELL. FROM FINÆUS.

was that of finding the length of a line neither end of which is accessible. (Figure XXVI.) Supposing the required distance to be the height of a tower, two observations were taken from points on the same level and in line with the tower, and the distance between the points of observation measured. The result was then calculated by the following rule: If both readings fall on the *umbra recta,* multiply the distance between the two stations by twelve and divide the product by the difference between the two readings. If both readings fall on the *umbra versa,* divide twelve by each reading in turn and take the difference of the quotients. Divide the distance between stations by this difference. If one reading fell on the *umbra recta* and the other on the *umbra versa,* it was necessary to reduce one reading to its equivalent on the other scale before making the calculation according to the rules. Here is an opportunity for a good algebraic problem. Pupils may verify the correctness of the fore-

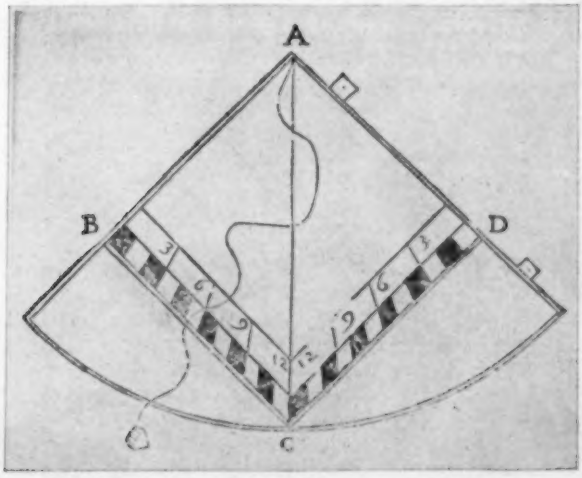


Figure XXII.

QUADRANT FORM OF GEOMETRIC SQUARE. FROM *Del Modo di Misurare* BY COSIMO BARTOLI, VENICE, 1689.

going rules by stating a pair of simultaneous equations and getting an expression for the height of the tower in terms of the three measurements.

A few of the additional problems mentioned by Finaeus in

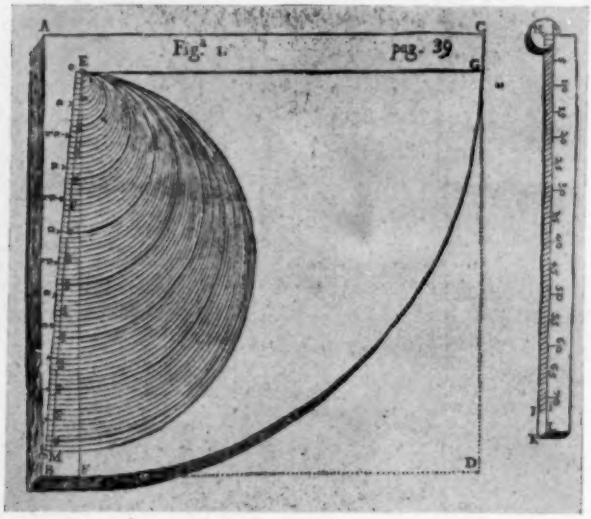


Figure XXIII.

INSTRUMENT FOR REDUCING MEASUREMENTS ON A SLOPE TO HORIZONTAL DISTANCES. FROM *Trattato dello Pratica di Geometria* BY LUDOVICO PERINI. THIRD EDITION, VERONA, 1751.

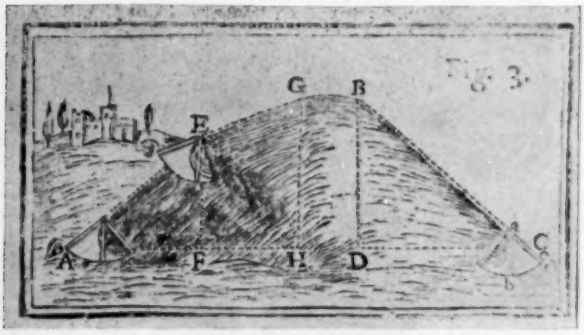


Figure XXIV.

FROM PERINI. (SEE FIGURE XXIII.)

connection with the geometric square are: (1) To measure a vertical distance from a higher elevation; (2) to measure a distance on a slope; (3) to measure the depth of a well.

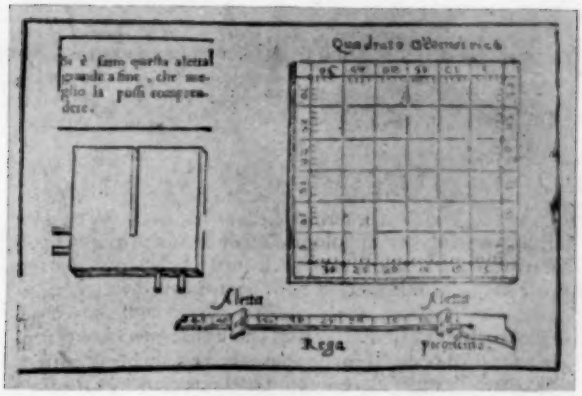


Figure XXV.

GEOMETRIC SQUARE DESIGNED TO AVOID CALCULATION. FROM BELLI.
(SEE NOTE UNDER FIGURE XII.)

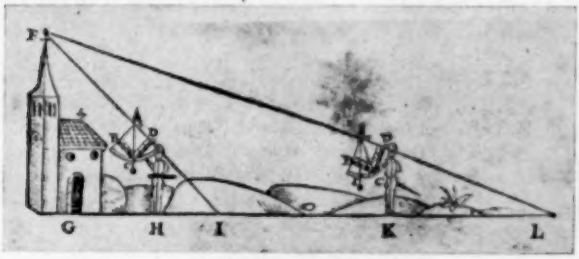


Figure XXVI.

MEASURING THE HEIGHT OF A TOWER WHOSE BASE IS INACCESSIBLE.
FROM FINAEUS.

(To be continued.)

CENTRAL SCIENTIFIC CO.

The Central Scientific Co., Chicago, are placing on the market a model of Brennan's mono-rail car. This model embodies the gyrostatic arrangement employed by Mr. Louis Brennan for the purpose of balancing a car upon a single rail, when the center of gravity of the car is above the rail. It is described by Prof. Perry in *Nature* for March 12, 1908, page 447, and by Prof. W. S. Franklin in the *Popular Science Monthly* for July, 1909, page 20. The design is that of Professors Crew and Tatnall of Northwestern University.

It is believed that many instructors in physics or engineering will find this car useful in illustrating vector multiplication, angular momentum, precession, and, indeed, all of Newton's laws of motion for the case of rotation. The manufacturers have an illustrated descriptive circular which will be sent free on request.

PROBLEM DEPARTMENT.

E. L. BROWN,

Principal North Side High School, Denver, Colo.

Readers of the magazine are invited to send solutions of problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott St., Denver, Colorado.

Algebra.

172. *Proposed by W. T. Brewer, Quincy, Ill.*

Find three square numbers which form an arithmetical progression.

Solution by Merton T. Goodrich, Dixfield, Me.

This problem is indeterminate, as the following treatment shows: Let the three numbers be x^2, $x^2+2ax+a^2$, and $x^2+2amx+a^2m^2$.

If they are in arithmetical progression, the difference between the first and second equals the difference between the second and third. That is, $2ax+a^2=2amx+a^2m^2-2ax-a^2$

Hence $x = \frac{a(m^2-2)}{2(2-m)}$, and $2ax + a^2 = \frac{a^2m(m-1)}{2-m}$.

and the three square numbers expressed in terms of a and m are:

$$x^2 = \frac{a^2(m^2-2)^2}{4(2-m)^2},\quad x^2 + 2ax + a^2 = \frac{a^2(m^2-2)^2}{4(2-m)^2} + \frac{a^2m(m-1)}{2-m},$$

$$x^2 + 2amx + a^2m^2 = \frac{a^2(m^2-2)^2}{4(2-m)^2} + \frac{2a^2m(m-1)}{2-m}.$$

From these equations it is seen that the values of the three square numbers depend entirely on the values which may be assigned to a and m. Now, m can not be 0, for then $x^2=x^2+2amx+a^2m^2$; neither can m be 1, for then $x^2+2ax+a^2=x^2+2amx+a^2m^2$; nor can m be 2, for then x is infinity. Yet even if we limit the square numbers to include only the squares of real positive integers not 0, an indefinite number of solutions is possible. For m may be given any value whatever, except 0, 1, 2, and a corresponding value assigned to a which will make x a real positive integer not 0, and we will have from the above equations three square numbers in arithmetical progression. Moreover, a may be multiplied by some number F, and since we may assign any real positive values not 0 to F, an indefinite number of solutions is possible for each value of m. However, if m receives certain values, such as 4, ½, etc., the square roots of some of our numbers can not be positive, although the square root of one of them x has been made positive.

The following is a list of a few of the numbers which meet the conditions of the problem, all those given being the squares of real positive integers. The square roots are denoted by x, $x+a$, and $x+am$. The corresponding values of m and a are also given.

m	a	x	$x+a$	$x+am$	Numbers.		
3	—2F	7F	5F	F	$49F^2$	$25F^2$	F^2
5/2	—4F	17F	13F	7F	$289F^2$	$169F^2$	$49F^2$
10/3	—12F	41F	29F	F	$1681F^2$	$841F^2$	F^2
8/3	—6F	23F	17F	7F	$529F^2$	$289F^2$	$49F^2$
7/3	—6F	31F	25F	17F	$961F^2$	$625F^2$	$289F^2$

This list may be continued indefinitely.

If x^2, y^2, z^2 be in Arithmetical Progression, then will $\frac{1}{y+z}, \frac{1}{z+x}, \frac{1}{x+y}$ be in Arithmetical Progression.

173. *Proposed by Orville Price, Denver, Colo.*

In how many ways can 10 red flowers and 5 white ones be planted around a circular plot so that two and only two red ones are adjacent? (From Hawkes' Advanced Algebra, p. 150).

I. *Solution by John P. Clark, Asbury Park, N. J.*

The five white flowers may be arranged about the circular plot in 4! ways. Between each two white flowers must come two red flowers, and these may be arranged in 10! different ways. The total number of arrangements is, therefore, 4!.10!.

II. *Solution by I. L. Winckler, Cleveland, O.; H. E. Trefethen, Kent's Hill, Me.*

The red flowers may be arranged in a circle in 9! ways. In each of these ways they may be separated into groups of 2 in 2 ways. Each of these 2 ways gives 5 places in which the white flowers may be placed. Therefore the white flowers may be arranged in these 5 places in 5! ways. Hence the total number of ways is given by $9!.5!.2 = 10!.4! = 87{,}091{,}200$.

Note by H. E. Trefethen.—In the above solution right-hand arrangements and left-hand arrangements are counted separately. If they are considered the same, the result is half as large.

Geometry.

174. *Proposed by Albertus Darnell, Detroit, Mich.*

If the sides of a quadrilateral are trisected and straight lines are drawn through the pairs of points nearest each vertex, a parallelogram is formed whose diagonals intersect in the centroid of the quadrilateral.

I. *Solution by I. L. Winckler, Cleveland, O.*

Let ABCD be the quadrilateral; HIJK the figure whose sides HI, IJ, JK, and HK are determined by points of trisection of the sides of the given quadrilateral nearest B, C, D, and A, respectively. Draw BD and AC and let O be their intersection. Suppose OC>OA. Take AF, on AC, equal to OC. Join F to E, the midpoint of BD. Trisect EF and let G be the point of trisection nearest E. Then, by the principles of mechanics, G is the centroid of ABCD. HI is parallel to AC, since it divides AB and BC proportionally. Similarly JK, IJ, and HK, are parallel, respectively, to AC, BD, and BD. Therefore HIJK is a parallelogram.

Let OA=a, OB=b, OC=c, OD=d. Take the diagonals AC and BD as axes of y and x, respectively, and O as origin.

Then the point I is $\left(-\frac{2b}{3}, \frac{2c}{3}\right)$, and K is $\left(\frac{2d}{3}, -\frac{2a}{3}\right)$.

$\therefore$ The equation of IK is,

$$y - \frac{2c}{3} = -\frac{a+c}{b+d}\left(x + \frac{2b}{3}\right).$$

Also F is the point $(o, c-a)$, and E is $\left(\frac{d-b}{2}, o\right)$.

$\therefore$ The equation of EF is

$$\frac{2x}{d-b} + \frac{y}{c-a} = 1.$$

Solving these equations for x we have

$$x = \frac{d-b}{3} = \frac{2}{3} \text{ of OE} . = x \text{ - coördinate of G.}$$

Therefore the diagonal IK passes through G, the centroid of the quadrilateral ABCD.

In the same way it may be shown that the other diagonal, HJ, passes through G.

II. *Solution by Editor.*

Use same notation as in solution I.

Let KJ intersect CD in N, and AD in R; let HI intersect AB in S, and BC in T. Join A and E, C and E. Q, the point of trisection of EC, nearest E, is the center of gravity of triangle CBD; P, the point of trisection of AE, nearest E, is the center of gravity of triangle ABD. G′, the center of gravity of the quadrilateral, will lie on the sect PQ. Let AO=$3y$, OC=$3x$, PG′=l. Then QG′=$x+y-l$.

By principles of Center of Gravity, $\frac{QG'}{PG} = \frac{ABD}{CBD'}$;

by Geometry, $\frac{ABD}{CBD} = \frac{AO}{OC} = \frac{y}{x}$.

$\therefore \frac{QG'}{PG} = \frac{AO}{OC}$, or $(x + y - l)\,x = ly$. Hence, $l = x$.

Let line through P and Q intersect IJ in L, and HK in M.

RP, PS, TQ, QN are parallel to BD, and each=⅔ of BE or ED. Therefore L is the midpoint of IJ, and M the midpoint of HK. Therefore IK and HJ intersect in midpoint of LM.

Now MP=⅓ AO=y. Therefore MG′=MP+PG′=$y+x$.

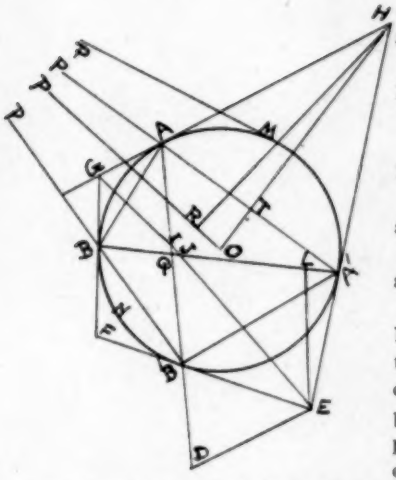
Hence G′ is the midpoint of LM, since LM=JK=2 $(x+y)$.

175. *By Request.*

Let PAA′, PBB′, PCC′ be three secants to a circle, PBB′ lying between the other two. Let the chords AB′, A′B intersect in Q, and the chords BC′, B′C intersect in R. Let M and N be the points of contact of tangents from P to the circle. Prove, *by elementary geometry*, that points M, Q, R, N are collinear.

Solution by Editor.

At the points A, A′, B′, B draw tangents to the circle, forming the circumscribed quadrilateral EFGH. Let O be center of the circle. Draw HR perpendicular to OP, EL parallel to FG, ED parallel to GH. Let GE, a diagonal of the circumscribed quadrilateral, intersect AB′ in I and A′B in J.



Since A′E and BF are tangents, angle EA′L=FBL.

Since EL is parallel to FB, angle FBL=ELA′.

∴.EA′L=ELA′, and EL=EA′.

Similarly, EB′=ED. But EA′=EB′, ∴. EL=ED.

Since triangles ELJ and BGJ are similar, EJ:JG=EL:GB.

Since triangles DEI and GAI are similar, EI:IG=ED:GA.

Since EL=ED, and GB=GA, ∴. EJ:JG=EI:IG. Hence I and J are the same point, and must therefore coincide with Q. Similarly, HF passes through Q. That is, the point of intersection of the diagonals of FGHE coincides with the point of intersection of the diagonals of AA′B′B.

Since angles HRP and HTP are right, the points H, T, R, P are concyclic. Therefore, OR.OP=OT.OH=$OA^2=r^2$; or $OR=\frac{r^2}{OP}$. If R_1 be the foot of a perpendicular from F to OP, then $OR_1=\frac{r^2}{OP}$. Therefore the points R and R_1 coincide, and HRF is a straight line.

Let R_2 be the foot of a perpendicular from M to OP. Since OMP is a right triangle, $OR_2 . OP=OM^2=r^2$ ∴. $OR_2=\frac{r^2}{OP}$. Therefore the points R and R_2 coincide. Hence M lies on the line HRF. Similarly, the point N lies on this line. But the point Q lies on the same line. Hence the points M, N, Q are collinear.

II. *Solution by G. B. M. Zerr, Philadelphia, Pa.*

Let AB, A′B′ intersect in E; BC, B′C′ intersect in F. Then QE is the polar of P; also RF is the polar of P. But MN is the polar of P. That is, QE, RF, MN coincide and are one and the same line. Therefore, M, Q, R, N are collinear.

Mr. Winckler furnished an elegant solution involving the principles of Harmonic Division.

Trigonometry.

176. *From School Visitor, June, 1894.*

Lines run from a point, P, within a triangular piece of land to the vertices A, B, C are 91, 102, and 80 rods, respectively; and a line 78 rods in length passing through the point P and terminating in the sides AC and BC, cuts off 3024 square rods adjacent to the vertex C. Find area of field.

Solution by W. B. Borgers, Grand Rapids, Mich.

Let line through P intersect BC in M, and AC in N. Let MP=s, Then PN=78—s. Let angle PCM=x, PCN=y, CPN=z.

$39\times80\times\sin z=3024$; therefore $z=75°45'4''$.

Then $\frac{PM}{80} = \frac{\sin x}{\sin (z-x)}$, or $s = \frac{80 \sin x}{\sin (z-x)}$ .....(1)

Similarly, $78 - s = \frac{80 \sin y}{\sin (z+y)}$ .....(2)

$\text{Sin NAP} = \frac{80 \sin y}{91}$.

$\text{Sin NPA} = \sin (z + y + \text{NAP})$

$= \frac{\sin (z+y)\,(8281 - 6400 \sin^2 y)^{\frac{1}{2}} + 80 \sin y \cos (z+y)}{91}$.

$\text{Sin MBP} = \frac{80 \sin x}{102}$. $\quad \text{MPB} = z - x - \text{MBP}$.

$\therefore \text{Sin MPB} = \sin (z-x) \cos \text{MBP} - \cos (z-x) \sin \text{MBP}$

$= \frac{\sin (z-x)\,(10404 - 6400 \sin^2 x)^{\frac{1}{2}} + 80 \sin x \cos (z-x)}{102}$.

$\text{NA} = \frac{91 \sin \text{NPA}}{\sin (z+y)}$, $\text{MB} = \frac{102 \sin \text{MPB}}{\sin (z-x)}$,

$\text{CN} = \frac{80 \sin z}{\sin (z+y)}$, $\text{CM} = \frac{80 \sin z}{\sin (z-x)}$.

Area $ABC = \frac{1}{2} CA \times CB \times \sin C$

$= \frac{1}{2}$ (CN+NA) (CM+MB) sin $(x+y)$.

The problem is indeterminate; but the triangle may be determined by assigning a value to x. Then y becomes known by (1) and (2).

For example, If $x = 30°$, Area = 10,182 sq. rds.

If $x = 0°$, Area = 8,114 sq. rds.

Remarks by G. B. M. Zerr, Philadelphia, Pa.

Let NM be the line 78 rods long with N in AC, and M in BC. The altitude of the triangle NCM is $77\frac{7}{13}$ rods. Sin CPM= $\frac{63}{65}$. Hence P can be any point on NM and every condition stated in the problem will still be fulfilled. Therefore there is an infinite number of triangles satisfying every condition stated, and the problem as stated is indeterminate. If the position of P on the line NM is given, in addition to what is already stated, the problem becomes determinate.

Mr. Edison Pettit, Peru, Neb., discussed the problem, and found area when MN is perpendicular to BC to be 9521.46 sq. rds.

Applied Mathematics.

177. *Proposed by H. C. Whitaker, Philadelphia, Pa.*

Four men are carrying a log 30 feet long, weighing 25 pounds per lineal foot, with a man weighing 150 pounds sitting on one end. Two of the men have hold of the log 5 feet from the end on which the man sits. At what point are the other two holding, provided each of the four men is carrying the same amount?

Solution by Arthur L. McCarty, Newark, Ohio.

Let x=distance of two men from end, A, opposite to that on which the man is sitting. Each man carries 225 pounds. Taking moments about A, we have—

$$450x + 450 \times 25 = 750 \times 15 + 150 \times 30.$$

$$\therefore x = 10 \text{ ft.}$$

CREDIT FOR SOLUTIONS RECEIVED.

Algebra 167. H. A. Morrison. (1)
Algebra 168. Merton T. Goodrich, H. A. Morrison, A. L. Womack. (3)
Geometry 170. John P. Clark. (1)
Applied Mathematics 171. W. J. Alexander, Merton T. Goodrich. (2)
Algebra 172. W. B. Borgers, W. T. Brewer (2 solutions), John P. Clark, Merton T. Goodrich, H. G. McCann, H. E. Trefethen, (6 solutions), E. E. Whitford, I. L. Winckler, G. B. M. Zerr. (15)
Algebra 173. W. B. Borgers, John P. Clark, H. E. Trefethen, I. L. Winckler, G. B. M. Zerr. (5)
Geometry 174. I. L. Winckler, G. B. M. Zerr. (2)
Geometry 175. I. L. Winckler, G. B. M. Zerr. (2)
Trigonometry 176. W. B. Borgers, Edison Pettit, G. B. M. Zerr. (3)
Applied Mathematics 177. W. B. Borgers, J. P. Drake, A. L. Ewing, H. G. McCann, A. L. McCarty, I. L. Winckler, G. B. M. Zerr. (7)
Total number of solutions, 41.

PROBLEMS FOR SOLUTION.

Algebra.

184. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

It is required to resolve the number $2x+y$ into two irrational factors, using the relation $2\ (a+b)^2+ab=(2a+b)\ (a+2b)$.

185. *Proposed by W. T. Brewer, Quincy, Ill.*

Within a rectangular garden containing one acre, there is a circular fountain, whose circumference is 40, 28, 52, and 60 yards distant from A, B. C, D, respectively, the four corners of the garden. Find the length and breadth of the garden, and the radius of the fountain.

Geometry.

186. *A sequel to No. 180.*

Points P, Q, R are taken on BC, CA, AB so that $\frac{BP}{PC}=\frac{1}{2}$, $\frac{CQ}{QA}=\frac{2}{3}$, $\frac{AR}{RB}=\frac{3}{4}$; the lines AP, BQ, CR form the triangle LMN; find the ratio of the areas of the triangles LMN and ABC.

187. *Proposed by Grace E. Shoe, Denver, Colo.*

ABCD is a quadrilateral; AC, BD its diagonals. If $AB\times CD+BC\times AD=AC\times BD$, it is required to prove that ABCD is a cyclic quadrilateral. (The converse of Ptolemy's Theorem).

Trigonometry.

188. *Selected.*

In any quadrilateral whose sides are a, b, c, d, prove that

$$d^2=a^2+b^2+c^2-2bc\cos\widehat{bc}-2ca\cos\widehat{ca}-2ab\cos\widehat{ab},$$

where $\widehat{bc}$ denotes the angle between the sides b and c.

Applied Mathematics.

189. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

To what depth does a log 20 inches in diameter sink in water, the specific gravity of the wood to the water being as 70 to 100?

REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.

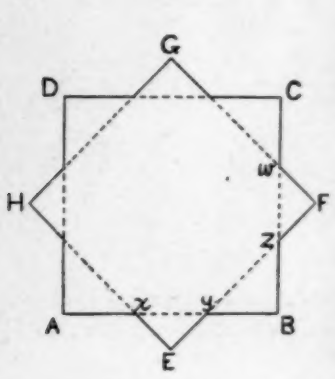
COMMITTEE ON INVESTIGATION: JAMES F. MILLIS, *Chairman, Francis W. Parker School, Chicago;* JOS. V. COLLINS, *State Normal School, Stevens Point, Wis.;* C. I. PALMER, *Armour Institute of Technology, Chicago;* E. FISKE ALLEN, *Teachers' College, New York, N. Y.;* A. A. DODD, *Manual Training High School, Kansas City, Mo.*

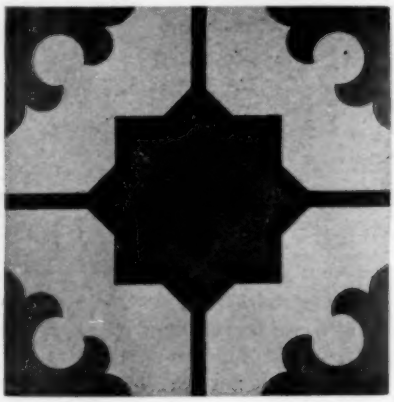
Teachers of mathematics who are interested in the movement to reform the teaching of mathematics in secondary schools by teaching the subjects more in relation to their practical uses are earnestly requested to coöperate with this committee (1) *by contributing real applied problems of algebra and geometry to be printed in these columns, where they will be accessible to all teachers for class room use,* and (2) *by using the problem material here printed in the class room with a view to determining its adaptability to the interests and needs of secondary school pupils. Criticisms of this material that are sent to the committee by teachers who have tried it in the class room will be of great value in carrying on the investigation.*

Problems.

By Mabel Sykes, South Chicago High School, Chicago.

1. The figure formed of two squares, superposed as in the drawing,





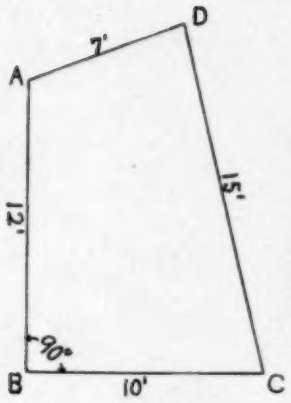
is frequently used in industrial design.

(a) If AB=12 and Ax=xy=yB=Bz=etc., find the area of the octagon $xyzw$..... and of the star AxEyBz.....

(b) Construct the figure so that Ax=xE=Ey=yB=etc., as shown in the accompanying oilcloth design.

By Arthur W. Belcher, Teachers College, New York.

2. The "gear" of a bicycle is the diameter (in inches) of a wheel whose circumference would equal the distance gone forward with one revolution of the pedals. Find it, if the diameter of the rear wheel is 28″ and the front and rear sprockets have 22 and 8 teeth, respectively.



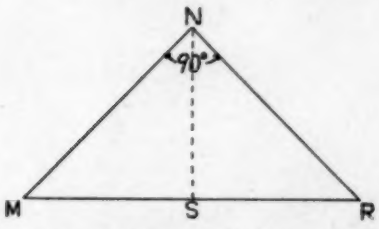
3. (a) Find the actual area of the sail in the diagram. (Hint: Draw AC.)

(b) Express the answer to (a) in yards of canvas, each 27″ wide.

(c) How many strips (running in the direction of the leech DC) of the above width would be required?

(d) Find approximate (to ⅛ of a yard) length of each strip, and total length required.

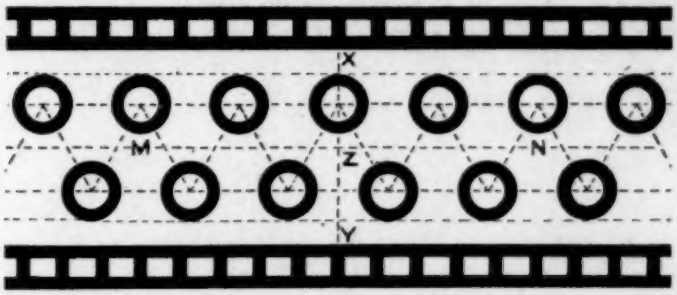
(e) If canvas costs $.25 a yard, and the other materials $1.00 altogether, how much will I save by making the sail myself instead of buying one for $9.50?



4. For a roof with right-angle pitch the carpenter's rule for finding length of rafter MN is to "add 5 per running foot" to length of MS. Test the accuracy of this rule and find resulting error to hundredths of feet, if MS=14′.

By R. S. Howlett, North Side High School, Denver, Colorado.

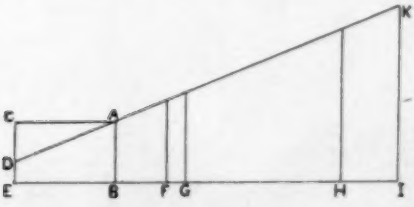
5. Embroidery pattern from Switzerland. Geometrical. XY ⊥ MN. Equal circles are described by pairs about the vertices



of equilateral triangles. The open spaces in the borders are equal rectangles, equally spaced. The material within the circles and the rectangles is cut out.

By G. E. Wright, Superintendent of Schools, Murray, Iowa.

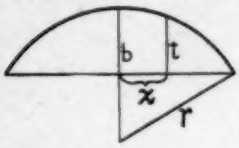
6. This problem was furnished by an abstractor. The notation has been changed from the original copy on account of the size of the cut.



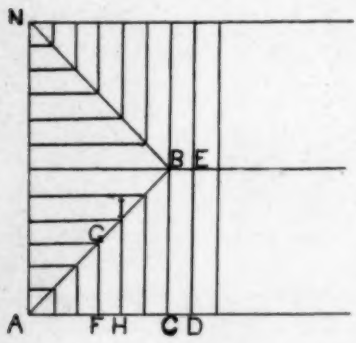
The drawing represents a plot of land divided as indicated. DE=22′ 8″, EB=100′, BF=50′, FG (Alley) =16′, GH=150′, HI=66′, and IK=232′ 6″. Find the length of AB in feet and the area of triangle ACD in square rods.

By C. I. Palmer, Armour Institute of Technology, Chicago.

7. When the arc of a segment of a circle is required for which the radius cannot be used, either by reason of its extreme length or because the center is inaccessible, it is desirable to obtain the arc without the use of the radius. This may be done by dividing the chord into a number of equal parts and calculating the several distances from the chord to the arc. Show that any distance $t=\sqrt{r^2-x^2}-(r-b)$, where the letters have values as shown in the figure. Given chord=20′ and b=4′, compute t for every 2′.

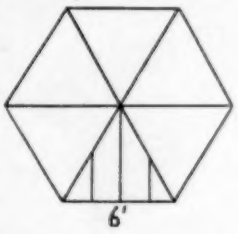


8. A carpenter uses the following theorem in drawing a regular octagon: In a regular octagon the semi-diagonal of the circumscribed square, having its sides coincident with four of the sides of the octagon, equals the distance along the side of the square from its corner to the remote vertex of the octagon occurring on that side of the square.



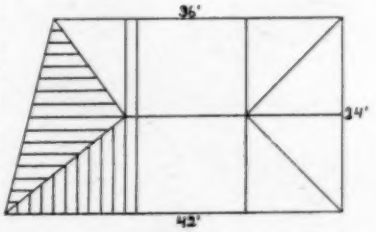
9. In the figure is a plan of one end of a roof of a house 18′ wide and 28′ long. CB, DE, etc., are common rafters; AB and NB are hip rafters; and FG, HI, etc., are jack rafters. Find the lengths to cut the several rafters, if the roof is ⅓ pitch. The rafters are 1′ 6″ from center to center, and are 2″ thick. The bevel on lower end of common and jack rafters is the same. Show how to find it by use of carpenter's square. Show how to find bevel of lower end of hip rafters. Of upper ends of common, hip, and jack rafters.

10. The figure is a plan of the roof of a hexagonal tower. Find lengths of rafters. The pitch is full, and the rafters are 1′ 6″ from center to center.

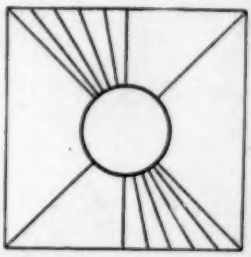


11. The figure is the plan of the roof of a building not rectangular in shape. Find lengths of the various rafters, and show how bevels are cut. Pitch one-third, and rafters 1′ 6″ from center to center.

12. The figure represents the plan of the roof of a house 20′

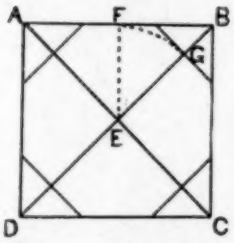


square, with a flat circular portion 8′ in diameter in the center. If the circular portion is ⅓ of the width of the building above the plates, and the rafters 2′ from center to center on the plates, find length of each rafter. Show how to find lengths by use of the carpenter's square.



13. In putting up blower pipes, two circular pipes 11″ and 14″, respectively, in diameter join and continue as a rectangular pipe 14″ in width. Find the length of the cross-section of the rectangular pipe.

14. Find the volume of a wash boiler if the bottom is in the form of a rectangle with a semicircle at each end. The rectangle is 10″ by 14″, with the semicircles on the smaller dimensions, and the depth of the boiler is 16″.



By James F. Millis, Francis W. Parker School, Chicago.

The following problems illustrate the type of problems that are suggested by books on carpentry.

15. To lay off an octagon on the end of a square piece of timber ABCD, draw diagonals AC and BD. With radius EF, draw arc cutting BD at G. Square out from G. Make a similar construction at each of the other corners of the square. Justify the rule.

16. By use of the steel square, lay out an angle of 45°.

17. By use of steel squares, lay out an angle of 60°.

18. To test the accuracy of a steel square, a thing which every carpenter should do, two squares are placed against some straight edge, as the edge of another steel square, and the heels brought together. Upon what principle of geometry is the test based?

19. The steel square may be tested also by measuring across from the 9-inch point of the tongue to the 12-inch point of the blade. If this distance is exactly 15 inches, the square is true. Why?

20. By use of the steel square, a circle may be drawn with any diameter AB as follows: Drive nails at A and B. Place the square with the blades against the nails, and keeping the blades constantly against the nails, slide the square around. A pencil held at the heel will describe a circle. Why?

21. To lay off the length of a brace with the steel square: Suppose that the post is four ft. and the beam 3 ft. Apply 3 times to the timber from which the brace is to be cut the distance across the square from the 12-in. point of the tongue to the 16-in. point of the blade. Why will this give the required length of the brace?

KANSAS ASSOCIATION OF MATHEMATICS TEACHERS.

The Kansas Association of Mathematics Teachers held its seventh annual meeting in Topeka on November fourth and fifth, the President, Miss Effie Graham of Topeka High School, presiding. The following program was given: "Suggestions as to the use of Bush and Clark in Plane Geometry" was discussed by W. V. Miller of Montgomery County High School, who had much to say in commendation of the text. He suggested that the number of propositions is smaller than that of our former text, that the group system is to be commended, that the originals are good and well graded, and that new terms and definitions are satisfactory. Discussion of the main paper brought out that all but two of the teachers perferred the new text to the old.

Miss Lucy T. Dougherty of Kansas City, Kansas, spoke on "Solid Geometry with Bush and Clark," in which she commended as especially worthy the treatment of polyhedral angles, of originals, of propositions on the surface of a sphere, of prismoidal formula, of volume of a sphere and sector and of the surface and volume of a cylinder, making some very useful suggestions for the use of polar triangles, prismoidal figures, and others which may be constructed in class.

Mr. C. H. Brooks of Cherokee County High School discussed the Geometry Report made to the Central Association of Science and Mathematics Teachers in 1908. Mr. Brooks for the most part upheld the report and pointed out the ways for teachers to use it.

It was brought out in general discussion of the report that a college man is not the one to write the text-books for the High Schools and that new "schemes" such as Unifying of Secondary Mathematics *may* not prepare the student for higher mathematics.

A symposium of seven subjects was given as follows:

1. Miss Olcott of Rosedale on the "Qualifications of an Up-to-date Mathematics Teacher" in which she showed that such a teacher should be up-to-date in other things, should attend Teachers' Associations, should read SCHOOL SCIENCE AND MATHEMATICS or other mathematical magazines, should have a college preparation, should try to hear leaders on Mathematics teaching, and should work out plans for himself.

2. Mr. Cook of Baker University spoke on "Amount of Daily Written Work Required on Algebra, and How to Credit It," in which he said there should be enough to fix the principle, and it should be credited relatively, one for a board recitation, one for a quiz, and one for the written work handed in.

3. Mr. Hall of Leavenworth, discussing "The Value of Formal Examinations," said the quiz or test has its place in checking both teacher and pupil, but the regular examination is necessary for an incentive or goal and for obtaining the three ends of Mathematics teaching, accuracy, rapidity, and neatness.

4. Miss Anna Speer of Atchison County High School said about "What Counts in Estimating Grades" that the test grade along with the teacher's own estimate was the fairest way to make up a grade.

5. Mr. W. C. Wheeler of Washburn College said "Fallacies and Puzzles" have their place in Mathematical instruction and are helpful

in keeping a student from becoming over confident in his logical proofs.

6. Prof. R. R. Price of the University of Kansas, speaking on "How may Mathematics be made a Part of the University Extension Course?" said the University expects to offer three grades of correspondence work in Mathematics: 1. High School Preparatory or Algebra and Geometry. 2. University and College grade consisting of College Algebra, Trigonometry, Analytics, and Differential and Integral Calculus. 3. Business course for Artisans, a year's course without University credit but of value to Artisans.

On November fifth Miss Edith Long of Lincoln, Nebraska, lectured on "The Unifying of Secondary Mathematics." Miss Long said in part that the objections to the new course were that teachers felt the need of a change but were unwilling to take the step, that Superintendents and Principals are often against a change, that there are no text-books for it, and that the necessity for Uniformity in large High Schools makes it much harder to try something new. Miss Long then proceeded to show how these objections have been removed in her own school. The pupils make their own text-books from day to day from the typewritten lessons. She outlined the course as consisting not simply of mixed mathematics but of problems which show an Algebraic and a Geometric side, while the check gives the Arithmetic side. The suggested beginning with shapes of things, leading to Geometric and Algebraic forms of the fourth fundamental operations and later to the Equation taken from complemental, supplemental, and vertical angles, and used in Geometric demonstration of them. Triangles and Quadrilaterals are treated in the same way.

Miss Long pointed out that her system does get results, and results which satisfy the University men.

Mr. Wagner of the Emporia High School spoke on what that school is doing in Unified Mathematics. He said they realized the course is not yet perfect but were well satisfied with their results. Their school is basing the first year on the equation.

Professor C. H. Ashton of the University of Kansas spoke on the Engineer's High School preparation, making a plea for putting the Algebra later in the course.

At the business meeting later the report of the nominating committee, naming Mr. A. M. Bogle of Kansas City High School for President and Miss Emma Hyde of Emporia High School for Secretary and Treasurer was adopted.

EMMA HYDE.

MORE LOCI PROBLEMS.

On page 884 of the December SCHOOL SCIENCE AND MATHEMATICS is found a list of easily accessible sources of loci problems. In reading this I was struck by a rather notable omission, namely, Halsted's Rational Geometry. In this is given a "collection of simple loci" numbering 20, followed by 45 loci problems, making a "content" not unworthy the attention of the earnest investigator.

Boston, Mass. LUCY B. UPHAM.

REPORT OF THE NINTH MEETING OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The Ninth Meeting of the Central Association of Science and Mathematics Teachers was held Nov. 26, 27, 1909, at the University of Chicago.

This meeting was the largest ever held by the Association, thirteen states being represented by more than four hundred in attendance. Not only is the Association becoming more widely known, but the increase in membership of nearly forty per cent during the past year indicates the interest in, and the appreciation of the value and permanence of its work. The fact that our members come from High Schools and Academies, Normal Schools, Colleges and Universities, each class being well represented, may be of interest.

The first meeting was held Friday morning in Mandel Hall, President Judson of the University of Chicago giving the address of welcome. His remarks were forceful and suggestive of lines of work profitable for our Association. The first address was given by Principal James E. Armstrong of the Englewood High School upon "Advantages of Sex Segregation in the High School." The experiment of limited segregation at Englewood High School has been in progress for nearly four years. Many important facts have been developed during this period. Mr. Armstrong represented the results of his investigation in a masterly manner. His clear, direct, convincing presentation of the subject held the close attention of the audience. After a short discussion, Professor Ernest H. Burton of the University of Chicago presented an address upon "Certain Features of China, Physical and Humanistic." His lecture, based upon his own observation in China, was of much interest. The absence of Prof. T. C. Chamberlin caused by his illness was deeply regretted, but his place was ably filled by Professor Burton.

The section meetings will be given more extended notice in separate reports to be published in this Journal. The work of each section was progressive and instructive. It was characterized by reports upon the fundamental principles and topics of Biology, Chemistry, Earth Science, Physics, and Mathematics. Many of these are of permanent value.

Friday evening the Annual Dinner, attended by 150 members, was held in Hutchinson Commons. Following this was the presentation of a report of a committee upon the "Relation of Elementary School Nature Study to Secondary School Science," by the chairman, Professor F. L. Charles of the University of Illinois. The report was presented in a printed outline, and, after careful explanation by the chairman, was thoroughly discussed from the floor.

The meeting was impressed with the importance of the subject, and a committee was ordered appointed to consider the question further and report at the 1910 meeting. The membership of this committee is as follows: Prof. F. L. Charles, University of Illinois; Prof. O. W. Caldwell, University of Chicago; Prof. J. A. Drushel, Teacher's College, St. Louis; Prin. W. M. Butler, Yeatman High School, St. Louis; Mr. Jas. H. Smith, Austin High School, Chicago.

The Executive Committee held its first meeting at 7:45 P. M., Nov. 25th, at the Great Northern Hotel. Eleven members representing all sections were present.

Much business of importance to the Association was discussed and action taken on several matters. A list of 101 names was presented by the Secretary with recommendation that the persons named be elected to membership. These were duly elected.

The request of the Executive Committee of the American Federation of Teachers of the Mathematical and the Natural Sciences that the Central Association increase its payment of dues from five cents to ten cents per member in accordance with the action of other associations, was acceded to..

The appointment of a general committee of five upon fundamentals of the work of the various sections to report at the 1910 meeting was ordered. This committee, as appointed by President Smith, follows: Biology—T. W. Galloway, James Millikin University, Decatur, Ill. Chemistry—A. L. Smith, Englewood High School, Chicago. Earth Science—Miss Marion Weller, State Normal School, DeKalb, Ill. Mathematics—D. A. Lehman, Goshen College, Goshen, Ind. Physics—W. E. Tower, Englewood High School, Chicago.

At the meeting Saturday morning much business of interest to the Association was accomplished. The first of this was the election of officers. The new officers of the Association are: President, James H. Smith, Austin High School, Chicago; Secretary-Treasurer, James F. Millis, Francis W. Parker School, Chicago; Assistant Secretary-Treasurer, V. D. Hawkins, Technical High School, Cleveland.

The Section Chairmen are: Biology, Dr. W. L. Eikenberry, School of Education, University of Chicago; Chemistry, Dr. Albert L. Smith, Englewood High School, Chicago; Earth Science, W. M. Gregory, Normal School, Cleveland; Physics, C. E. Spicer, Joliet High School; Mathematics, C. A. Petterson, Jefferson High School, Chicago.

The reports of the Secretary and Treasurer were read and accepted upon the favorable report of the Auditing Committee.

The committee on resolutions reported as follows:

Resolved, That the Ninth Annual Session of the Central Association of Science and Mathematics Teachers hereby expresses its thanks and appreciation to the University of Chicago and to the Reynolds Club for their hearty reception and cordial entertainment of this Association and its members, to Principal James E. Armstrong, and to Dr. Ernest T. Burton for their interesting and inspiring addresses, to all who have contributed to the success of the general and sectional programs; and to the Local Committee of the University of Chicago for their efficient work in advertising this meeting.

That we express our regret and sympathy for Professor Thomas Chrowder Chamberlain in the illness which prevented his attendance upon our meetings.

That we tender Willis E. Tower the thanks of the Association for his efficient and untiring services as Secretary-Treasurer during the past two years, and regret his decision to retire from office.

That we offer thanks to the Publication Committee, especially to the Chairman, Mr. Chas. H. Smith, for its arduous and very satisfactory work in preparing the volume of "Proceedings for 1908."

FRANKLIN T. JONES,
AMELIA MCMINN,
H. E. COBB,
Committee.

It was unanimously voted to continue the publication of the Proceedings of the Association. The value of the Proceedings has been shown especially in the increased membership and interest in the Association by those who can rarely be present at its meetings.

The question of Institutional membership in the Association was brought up by Professor C. R. Mann, and it was voted that the Executive Committee look into the matter and take such steps as seem advisable.

Mr. V. D. Hawkins of Cleveland presented a cordial invitation to the Association to hold the 1910 meeting in that city.

This invitation was supported by letters from Mayor Tom L. Johnson, President Charles E. Howe of Case School of Applied Science, Superintendent Elson of the Cleveland schools, and the Northeastern Ohio Association.

This invitation so strongly impressed the meeting that it unanimously recommended to the Executive Committee favorable consideration of the invitation. The Executive Committee has in consequence unanimously voted to hold the 1910 meeting in Cleveland.

Upon motion by Professor C. R. Mann it was voted that the Executive Committee appoint a committee of five to take up the matter of scientific investigation in education with special reference to methods of testing results to report at the 1910 meeting of the Association.

The business meeting was then adjourned, the section programs being taken up.

Saturday afternoon three groups of members visited the Fisk Street Station of the Commonwealth Electric Company and the New Steel Mills at Gary, Ind.

TREASURER'S REPORT.

Nov. 27, 1908-Nov. 24, 1909.

RECEIPTS.

Balance shown by last report	$ 400.00
Dues 311 members at $2.50	777.50
Dues 1 member at $2.25	2.25
Dues 51 members at $2.00	102.00
Dues 1 member at $1.50	1.50
Dues 29 members at $1.00	29.00
Dues 57 members at $.50	28.50
Sale of dinner tickets November 27, 1908	59.00
Advertisements in 1908 Program	117.00
Advertisements in 1909 Program	58.00
2 copies Correlation Reports	.50
17 copies Proceedings at $.50	8.50
	$1,183.75
	$1,583.75

EXPENDITURES.

Publishers, SCHOOL SCIENCE AND MATHEMATICS		$ 545.25
Expenses, 1908 Meeting.		
Badges	$10.40	
Professor Duncan	65.00	
Mark Jefferson	5.00	
Ushers, lunches, etc.	5.25	85.65
Expense, Section Officers, 1908		20.51
Expense, Secretary-Treasurer.		
Bond	$10.00	
Postage and Supplies	45.59	
Assistance	6.80	62.39
118 Dinners at fifty cents		59.00
1908 Program, printing and distribution		158.30
1908 Proceedings, printing and distribution		365.43
American Federation of Teachers		17.50
Refund to Iowa and Indiana Associations		1.50
Refund to Macmillan and Co.		10.00
Post cards, letter heads, circulars		32.93
Mathematical Reprints		5.50
Exchange on checks		1.20
		$1,365.16
Balance, Cash on Hand		218.59
		$1,583.75

WILLIS E. TOWER, Treasurer.

MEMBERSHIP REPORT.

FOR YEAR ENDING NOV. 24, 1909.

Paid up membership, Nov. 27, 1908	324
Honorary membership, Nov. 27, 1908	4
Total membership, Nov. 27, 1908	328
New members added during year	161
Renewals, counted as expired last year	47
Total	536
Resigned and dropped from list	40
No. names on list Nov. 24, 1909	496
Expired, kept on list as per constitution	47
Total membership, Nov. 24, 1909	449
Honorary membership, Nov. 24, 1909	4
Paid up membership, Nov. 24, 1909	445
Increase during year	121

WILLIS E. TOWER, Secretary-Treasurer.

The attendance of the Columbia University Summer School increased from 1,498 in 1908 to 1,936 in 1909. There was an increase of 50% in the number of students taking courses in physics.

REPORT OF THE MEETING OF THE MATHEMATICS SECTION C. A. S. AND M. T., NOVEMBER 25, 26, 1909.

The section held two sessions, one Friday afternoon and one Saturday morning. The preliminary report of the committee on Real Applied Problems in Algebra and Geometry was discussed Friday afternoon. The second report of the committee on the Unification of Secondary Mathematics was discussed Saturday morning. Time was given at the close of both sessions to reports of the work of the International Commission on the Teaching of Mathematics.

The chairman, Mr. Chas. Ammerman of St. Louis, presided. The following business was transacted:

It was decided that the committee on Real Applied Problems in Algebra and Geometry be retained for another year.

At the Friday afternoon session a committee was appointed to consider the question of reprints of the problems that have been published or are to be published in SCHOOL SCIENCE AND MATHEMATICS and that have been collected through the investigations of this committee. Mr. J. V. Collins of Wisconsin was made chairman of this committee. This committee reported on Saturday morning that copies of all problems that have appeared to date could be obtained from the publishers of SCHOOL SCIENCE AND MATHEMATICS at the rate of 5c a copy, provided that a sufficient number of copies were guaranteed. The report was accepted and the required number of copies were taken by members of the section in a few moments.

It was decided that a committee of five be appointed to consider the question of collecting, tabulating, and reporting upon data with respect to any departures from established traditions in mathematics teaching in the territory covered by this association; and that the chairman of this committee be the member from the mathematics section of the similar committee appointed by the Central Association.

It was decided that an assessment of 25 cents be levied to carry on investigations in the section next year, and that this assessment be voluntary. Thirteen dollars was collected.

It was decided that a committee be appointed to meet with similar committees from the physics and chemistry sections to consider questions of notation in mathematics.

The section elected the following officers for the ensuing year:

C. A. Petterson, *chairman*, Jefferson High School, Chicago; R. L. Short, *vice-chairman*, Technical High School, Cleveland; Mabel Sykes, *secretary*, South Chicago High School, Chicago.

PRELIMINARY REPORT OF THE COMMITTEE ON REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.[1]

The preliminary report of the committee on Real Applied Problems in Algebra and Geometry was presented by Mr. J. F. Millis of Chicago, chairman of the committee, who called attention to the main features of the report.

[1]This report was printed in the November number of SCHOOL SCIENCE AND MATHEMATICS.

The discussion was opened by Mr. G. A. Harper of Wilmette, Ill. This paper will appear in a future number of this Journal.

Mr. J. A. Foberg of Chicago: The scope of the report and the attitude of the committee are most gratifying. The general use of these problems must in time necessitate two changes in the curriculum: (1) a change in the arrangement of mathematics courses and (2) a simplification of the material presented. Credit should be given in the list of references to a typewritten pamphlet by Professor C. I. Palmer of Armour Institute that was privately circulated a few years ago.

Mr. C. W. Newhall of Minnesota: The work of the committee is most timely and valuable and their report should be praised for the moderation and reasonableness of its recommendations. A word of caution to teachers may be well, however. Mathematics has a value for its own sake. Care should be taken not to cut the heart out of it in attempts to make it more interesting.

Mr. J. V. Collins of Wisconsin, Mr. C. I. Palmer, and Dr. H. E. Slaught of Chicago called attention to the magnitude of the work undertaken by the committee in general and by the chairman in particular, and strongly urged everyone present to assist in the search for problems and to send the same promptly to the committee without further notice. Members of the section also called attention to the fact that pupils in mathematics classes would often bring in interesting problems when properly encouraged.

Mr. F. Hester of Minnesota: The introduction of the definitions of sine, cosine and tangent and of computations of heights and distances as practical problems when studying proportion is interesting and valuable.

Mr. C. E. Comstock of Peoria, Ill.: The report needs revision to determine how far the problems collected are applicable to class room use. Another question of importance is the place of the practical problem in the mathematics course. If it is made the hub about which all else revolves no unity is possible. Problems that are not practical are often interesting.

Mr. Hanstein of Chicago: The use of practical problems is the only way to enforce the teaching of construction drawing and of geometry.

Mr. H. C. Harvey of Missouri: The closer articulation of the various branches of mathematics and of mathematics with mechanical drawing and manual training, should be insisted on.

Mr. Jocelyn of Michigan: Practical problems are good and the teacher should be familiar with them that the attention of the class may be called to all applications of the lesson from day to day.

The Second Report of the Committee on the Unification of Secondary Mathematics.

Mr. H. E. Cobb of Chicago, chairman of the committee, presented the report. The report consisted of statements from members of the committee who had been at work on special phases of the subject. Correspondence with associations of teachers of mathematics throughout

the country showed that although little or nothing definite had been done in the line of unification in these associations, there was more or less interest in the subject which was likely to result in something definite in the near future. The report contained a list of schools where efforts at unification were in progress, it discussed the tentative outline of a four years' course given in the preliminary report of a year ago, and gave some account of the unified course in mathematics given at Lewis Institute, Chicago.

Mr. R. L. Short of Technical High School, Cleveland, Ohio: A course in unified mathematics is given to half the first year pupils in the Technical High School at Cleveland. This course includes number study, algebra, and geometry. No apparatus is used but ruler and compasses. The relations of algebra to number study are particularly interesting and profitable. Little is done with real applied problems in either the first or second year. Later special courses in real problems are given to segregated classes. Problems in domestic science and home economy are given to the girls. Real problems are to a large extent not feasible except to classes that are taking or have taken the necessary shop work. When an explanation is necessary the practical nature of the problem disappears.

Owing to the lateness of the hour Mr. H. C. Wright of Clyde, Ill., was at his own request excused from the reading of his paper.

This paper will be published in a future number of this Journal.

Dr. H. E. Slaught of Chicago: Three kinds of real problems appear to be necessary—one for mixed classes, one for boys that take manual training, and one for girls that have taken domestic science. The great question is to find problems for mixed classes. No one has yet mentioned problems taken from architecture and design which are admirably suited to this purpose.

The Work of the International Commission on the Teaching of Mathematics.

Dr. J. W. A. Young, representative of the American commissioners: The commission was appointed at the International Congress of Mathematicians in Rome, 1908. In America fifteen committees have been appointed covering all the work. Half of the report will be devoted to instruction as it is, half to proposed changes. It must be remembered that the representation of instruction as it is, is partly for the benefit of foreign countries. Progress has been made in all countries on the report. America has made the most detailed organization. The work in Germany is the most comprehensive. Here there are seventeen preparatory reports. Two of these—one on the teaching of mathematics to girls and one on teaching from present texts, have been printed. This latter report is very comprehensive and is illustrated with graphs.

Mr. I. S. Condit, chairman of subcommittee on Preparation of Teachers of Mathematics for Seventh and Eighth Grades of Public and Private Elementary Schools: The committee consists of five members. Each member is assigned a group of states and in that

group is investigating 100 public schools in cities of over four thousand population. The inquiry in each section also includes public and private normal schools, city training schools, and departments of education in colleges and universities.

The purpose of the committee is to ascertain (1) present conditions, (2) future tendencies. In the questionnaire for public and private elementary schools some special points are (*a*) number of teachers in seventh and eighth grades, (*b*) preference as to sex of teachers, (*c*) nature of courses of study in mathematics, (*d*) preparation of teachers, (*e*) difficulties in teaching seventh and eighth grade mathematics and ways of overcoming same.

Schools for training teachers are asked to give information on such points as (*a*) attendance and plan of organization, (*b*) nature of course, (*c*) opportunities for practice teaching of mathematics, (*d*) time given to history and teaching of mathematics, (*e*) relative value of training in subject matter and training in method.

The response from the section assigned to the chairman has been prompt and generous, about fifty per cent being returned within two weeks. Time has been too short for getting reports from other sections. The enclosing of a stamped, self-addressed envelope has aided in securing returns.

Mr. H. C. Harvey of Kirksville, Mo., reported on the work of the subcommittee on State Normal Schools.

Mr. C. E. Comstock of Peoria, chairman of subcommittee on Scope and Arrangement of the Mathematical Curriculum in Elementary and Secondary Schools: It will be necessary that this committee review the work of the first seven committees. The committee will report upon and discuss the arrangement of courses in arithmetic, algebra, and geometry, and the various phases of unification, the results of experiments in unification and correlation including difficulties encountered and successes and failures, and the dangers and results of the pressing down of algebra and geometry from colleges into the secondary schools.

MABEL SYKES, Secretary.

THE SOUTHERN CALIFORNIA SCIENCE AND MATHEMATICS ASSOCIATION.

The first general meeting of this association since its formation by the affiliation of the Southern California Association of Mathematics Teachers and the Southern California Science Association, was held at the Polytechnic High School, Los Angeles, on Saturday, December 4, 1909.

The attendance totalled seventy-seven for the entire meeting.

The matter of joining the American Federation was left to the Executive Committee with power to act.

The question of the formation of sections received favorable consideration and a meeting of those interested in Earth Science resulted in the formation of a section, with provision for a meeting during the latter part of January at Occidental College, when Prof. W. A.

Fiske will read a paper. Other sections in Chemistry-Physics, Biology, and in General Science will be formed, a special meeting being called for each at which a definite organization will be effected.

An invitation from the Los Angeles High School to hold the April meeting there was accepted, subject to final arrangements by the Executive Committee.

Officers for the ensuing year were elected as follows: President, H. T. Clifton, Throop Polytechnic Institute, Pasadena; Vice-President, W. H. Snyder, Hollywood; Secretary-Treasurer, G. C. Bush, South Pasadena.

H. T. Clifton, Secretary.

NEW YORK SECTION OF THE ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

The first meeting of the New York Section of The Association of Teachers of Mathematics in the Middle States and Maryland was held Friday evening, Nov. 12th, at the High School of Commerce, New York City. The topic for the evening was "Mathematics for Service or for Culture." "Mathematics for Service" was presented by Dr. Ernest R. van Nardroff, Principal of the Stuyvesant High School, New York City. Dr. van Nardroff spoke from the experience of an able physicist, and offered a high school course of study, including the topics of algebra, geometry, trigonometry, analytical geometry and the calculus, which are useful in physics. "Mathematics for Culture" was presented by Dr. William H. Metzler, Professor of Mathematics, Syracuse University, Syracuse, N. Y. That Dr. Metzler spoke convincingly, or that the opinion of the meeting was already formed was evidenced by the discussion that followed. This discussion was animated and largely in favor of pure mathematics for its own sake.

Lao G. Simons, Secretary.

ARTICLES IN CURRENT MAGAZINES.

Astrophysical Journal for November: "Redetermination of the Wave-Lengths of the Arc and the Spark Lines of Titanium, Manganese, and Vanadium; The Effect of Capacity and Self-Induction on the Wave-Lengths of the Spark Lines," Clinton M. Kilby; "The Spectrum of Comet 1908 c," H. Rosenberg; "On the Absorption of Light in Space," J. C. Kapteyn.

The Auk for October: "A Nesting of the Blue-winged Warbler in Massachusetts," Horace W. Wright; "An Inquiry into the History of the Current English Names of North American Land Birds," Spencer Trotter; "A Study of a Breeding Colony of Yellow-headed Blackbirds: Including an Account of the Destruction of the Entire Progeny of the Colony by Some Unknown Natural Agency," Thomas S. Roberts; "Birds of Central Alberta," Sidney S. S. Stansell; "The Birds of Colorado," Wells W. Cooke.

Botanical Gazette: "Some Fungus Parasites of Algae," George F. Atkinson; "Influence of Electricity on Microörganisms," George E. Stone; "Cytology of Cutleria and Aglaozonia," Shigeo Yamanouchi.

Conservation for December: "Forestry in Japan," J. Barrington Moore; "The South's Concern in the Appalachian Project and How to Make Its Influence Felt," John H. Finney; "A Beginner in Forestry," Anne Warner (Paper 6); "Theodore Roosevelt, Dynamic Geographer," Frank Buffington Vrooman.

Economic Geology for October-November: "On the Origin of Petroleum," Leonard V. Dalton; "Problems in the Geology of the Havraki Goldfields, New Zealand," A. M. Finlayson; "The Classification of Geologic Materials," Frank F. Grout; "The Classification of Coal," Frank F. Grout; "Ore Deposition at Aspen, Colorado," G. F. Loughlin.

Journal of Geology for October-November: "Physical Geography of the Pleistocene with Special Reference to the Pleistocene Conditions," Rollin D. Salisbury; "Paleogeographic Maps of North America," Bailey Willis; "Origination of Self-Governing Matter and the Influence of Aridity upon Its Evolutionary Development," D. T. MacDougal; "Description of a Permian Crinoid Fauna from Texas," Stuart Weller; "The Molaspina Glacier Region of Alaska," Lawrence Martin.

Mining Science for December 2, 1909: "Determination of Depths to Horizontal Coal Seams," Arthur Lakes; "Radioactivity of the Camolite of Southwest Colorado."

Monthly Weather Review for June, 1909: "Annual Rise of the Columbia River," E. A. Beals; "Frost Damage Prevented by Covers," A. G. McAdie; "The Fireball of September 20, 1909"; "Weather Cycles in the Growth of Big Trees," A. E. Douglass; "Squalls and Thunderstorms," J. Loisel.

Photo-Era for December: "The R. P. S. Exhibition at London," E. O. Hoppé; "The Autochrome Plate in Its Relation to the Color-Theory of Young and Helmholtz," Fred D. Maisch; "Diffusion of Focus in Enlargements," F. J. Mortimer; "Improving Negatives of Uneven Density," Phil M. Riley; "Admiring Bad Pictures," William Howe Downes.

Physical Review for December: "On the Electric Charges Acquired by Insulated Potassium Salts and Other Radioactive Substances in High Vacua," J. C. McLennan; "The Heat of Dilution of Zinc Amalgams," W. D. Henderson; "The Secular Softening of Hard Steel," C. Barus; "Dispersion of Electric Double Refraction and Ordinary Dispersion in Liquids," Harold E. McComb; "Dispersion of Magnetic Double Refraction in Liquids Compared with that of Electric Double Refraction," C. A. Skinner.

Popular Science Monthly for December: "The Planet Venus," Dr. Percival Lowell; "The Argument for Organic Evolution Before the 'Origin of Species,'" Arthur O. Lovejoy; "An Arraignment of the Theories of Mimicry and Warning Colors," Abbott H. Thayer; "What Pragmatism Is, as I Understand It," Thomas Mitchell Shackleford; "Immigration and the Future American Race," Dr. Albert Alleman; "Environment and Productive Scholarship," Dr. W. J. Humphreys; "Medieval Creation Myths," B. K. Emerson.

Technical World Magazine for December: "The War with the Mountains," Henry M. Hyde; "Air-Ship Flight that Startled Europe," Dr. Alfred Gradenwitz; "Train-Dispatching by Telephone," Edward I. Pratt; "When the Soil Gets Tired," René Bache; "Killing Ticks Ends Cattle Fever," Robert Franklin; "Making the High School Democratic," H. G. Hunting; "Why Mines Explode," William Atherton Du Puy.

BOOKS RECEIVED.

Proceedings of the 13th Annual Meeting of the New York State Science Teachers' Association. 138 pages. University of the State of New York.

Proceedings of the 24th Annual Meeting of the Associated Academic Principals of the State of New York. 144 pages. University of the State of New York.

Constructive Drawing, by Herman Hanstein, director of drawing Lane Technical High School, Chicago. Illustrated. Keuffel and Essex Company, Chicago.

A History of the Teaching of Elementary Geometry, by Alva W. Stamper, State Normal School, Chico, Cal. Pp. 163. New York: Teachers College, Columbia University. Price, $1.50.

BOOK REVIEWS.

Elementary Algebra, by C. H. French, M.A., and G. Osborn, M.A., Mathematical Masters at the Leys School, Cambridge. 506 pages. 1908. Cambridge: at the University Press.

In a revised and enlarged form this Elementary Algebra includes common logarithms and the binomial theorem for positive integral exponents. It is said to cover the ground of all school examinations not requiring a special knowledge of the subject, and to be amply sufficient for such examinations as the London Matriculation, the Cambridge Previous, and others of similar standard. Evidently the authors intended to write a book for boys who are undergoing a rigid course of mental training, and who are to pass a more or less difficult examination in pure mathematics. At any rate the book shows no trace (except, possibly, in the graphical work) of the reforms urged by Professor John Perry and an increasing number of English mathematics teachers.

H. E. C.

Constructive Drawing, by Herman Hanstein, Lane Technical High School, Chicago. 140 exercises, 38 plates. Keuffel and Esser Company, Chicago.

This is a splendid book, compiled by one who is more than a past master of the subject. Those who know anything about constructive drawing acknowledge this to be one of the most helpful books of the kind ever published. The name of the author is enough to recommend its adoption and use anywhere. The book is the result of thirty years of experience in office, shop, and drawing room. The exercises are selected for their practical application for those who expect to follow architectural, mechanical, or engineering callings. For the person who is trying to become proficient in constructive drawing, privately, there is no better help. The drawings for illustrative purposes are well selected and perfectly executed. There is a complete index. Several pages are devoted to description of drawing instruments and different styles of letters. The book is six by eight inches and is well bound in cloth. It deserves a wide circulation.

C. H. S.

Standard Vest-pocket English-Italian Italian-English Dictionary. Vest-pocket size. 528 pages. Leather, gilt edges, double index. 75c. Cloth, red edges, double index, 50c. Laird & Lee, Chicago.

This is a timely little book, and will be found an invaluable help to all who wish to learn to speak the Italian language as well as to the large and constantly increasing Italian population in America in their efforts to learn English.

The work is carefully compiled, edited and arranged, and contains accurate pronunciation tables and abridged compendiums of the grammars of both languages, thus making it a valuable aid to students as well as to the general public. The work is most artistically bound.

A.

The Pupils' Arithmetic, Primary Book, Part Two. By James C. Byrnes, Julia Richman and John S. Roberts, New York City. Pp. 218. Price, 30 cents. 1909. The Macmillan Company.

In this book which covers the work of the fourth year multiplication and division are made the *core* of the work. The authors direct attention to the following features: (1) The large number of exercises in pure arithmetic. (2) The large number of problems in applied arithmetic. (3) The careful grading of exercises, the avoidance of long numbers and lengthy processes, and the regular alternation of exercises in abstract number and concrete problems. (4) The model solutions. (5) The variety and interesting character of the problems.

H. E. C.

Economic Zoölogy. By Herbert Osborn, M.Sc., Professor of Zoölogy and Entomology in the Ohio State University. 490 pages, 269 figures. Price, $2.00. 1908. The Macmillan Company.

Economic Zoölogy, by Professor Osborn, is a text-book for schools and colleges. It is economic in treatment only to the extent that the economic relations of each group are discussed, if there are any. This means that the book is not constructed on economic lines at all, but after the fashion of the conventional text-book in zoölogy. To begin with Protozoa and finish with Mammalia is to do what many have done before who gave no thought to the economic phases of animal life. It would have been interesting to examine a text-book on economic zoölogy.

Probably all consumers of books will agree to the proposition that no text-book should be written unless there is some positive contribution made to the subject itself or to the teaching of it. This book ventures to be new only in the unusual space it gives to the effect many animals have directly or indirectly on the welfare of man. For this we give it due credit.

There is another proposition to which many will agree: Text-books in all subjects should be written in good rhetorical English. There is constant occasion in the exposition of the morphological characteristics of a group to make use of sharp, clear cut distinctions. For example, the author of "Economic Zöology" undertakes to distinguish the *hydrula* and the *scyphula* type of cœlenterate. He describes the latter (p. 44) and leaves the structure of the former to be inferred.

which may not be done correctly by the student. The next sentence in the paragraph to the attentive student would almost certainly be taken to refer to the scyphula, but it really refers to a characteristic of all cœlenterates. Another extremely common fault in the writing is the failure to explain a term, such as "ectoderm" when it is first used. There is no system whatever in the book for the explanation of terms. The meanings come casually and in unexpected connections.

The bad pedagogy of giving a statement of the general characteristics of a phylum before any exposition of the characteristics of a particular species is made is much older than "Economic Zöology." But there are many other pedagogical "finds" that are equally slow in making the rounds. It is a waste of pictures to print them without bringing them into the unity of the book by judicious and frequent references. It is also bad economy to print pictures that have worn down black with the use of a generation of printers, in possibly a score of text-books.

The topics of the final chapter constitute another bit of evidence indicating that the author has failed to organize his work in telling fashion. Is it not a little late in that place to discuss Distribution, Adaptation, Evolution, Animal Industries and Geological Succession?

H. R. L.

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

The American Commissioners earnestly request all to whom questionnaires have been addressed to send in their replies at the earliest possible moment. The set of reports that are being prepared in this country should show the work that is being done here, and to have these reports ready when those of other countries are presented, a great deal of hard work must be done this winter, and prompt replies to the questionnaires are indispensable.

The third annual meeting of the Illinois State Academy of Science will be held at the University of Illinois, Urbana, February 18-19, 1910.

ERRATA.

The second cut on page 916 of the December number should have been placed in connection with the article on "Direct Demonstration" on page 918.

In line 41, page 920, for $s=\sqrt{\tfrac{1}{2}(a+b+c)}$ read $s=\tfrac{1}{2}(a+b+c)$.